CONFINED FLOW UNDER WEIRS RESTING ON ANISOTROPIC AND NON-HOMOGENEOUS POROUS MEDIA

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CERTIFICATE

Certified that this work "Confined Flow Under Weirs Resting on Anisotropic And Non-Homogeneous Porous Media" has been carried out by Sri Satyendra Kumar Sinha under my supervision and the same has not been submitted elsewhere for a degree.

A.J. Valsangkar
A.J. VALSANGKAR

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SATYENDRA KUMAR SINHA

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NOTATIONS

Symbol	Description
b	Width of the weir
b_1	Distance of sheet pile from heel of the weir
c	constant
d	Distance of horizontal joint from the base of the weir
d'	Distance of lower pervious or impervious boundary from heel of the weir
h	Difference in total heads at upstream and downstream boundary.
l	Distance of vertical joint from heel
K	Coefficient of permeability
m	Distance of inclined pervious boundary from toe.
P	Pressure
S	Embedded length of vertical sheetpile
S'	Embedded length of inclined sheet-pile

Symbol	Description
x	Variable distance along base of the weir
z	Elevation head
α	b/s
α'	b/s'
β	m/b
η	l/b
γ	Angle in degree maded by embedded length of sheet pile with the base of weir.
λ	d/b
λ'	d'/b
μ	b_1/b
ϕ	Velocity potentional
θ	Inclination of flow boundary

ABSTRACT

In many instances one encounters with the problem of flow through non-homogeneous and anisotropic porous media. In general, for analysing such flow situations either numerical or analog methods are used. The present study deals with the analog solutions for the problem of confined flow below the weir resting on non-homogeneous and anisotropic soils.

The non-homogenity considered in the present analysis corresponds to the jointed rock mass which is characterised by two permeabilities, namely primary and secondary permeabilities corresponding to rock material and joints, respectively. The analysis is presented for a weir with a vertical sheet pile resting on an infinite rock mass considering one major joint either to be vertical or horizontal. The influence of (i) disposition of sheet pile (ii) length of sheet pile, (iii) depth at which horizontal joint is located & (iv) the presence and location of vertical joint, on the equipotential lines and the uplift pressure is investigated. The results clearly bring out the necessity of considering the disposition of joint in the analysis of such a flow situation.

Similar to non-homogenity, the anisotropic nature of the soil also governs the confined flow behaviour considerably. In general, by transformation technique, the anisotropic flow domain can be converted into an equivalent isotropic region for determination of uplift pressure, exit gradient etc. Taking the general case of anisotropy and varying the lower flow boundary condition, analog solutions have been obtained for a weir with one sheet pile. The influence of (i) the location of sheet pile (ii) inclination of major axis of ellipse of anisotropy with respect to horizontal direction, (iii) lower flow boundary conditions (either highly pervious or impervious) is investigated in detail.

As it is very difficult to obtain the solutions for a two layered soil system which is not parallel to horizontal direction, it is hoped that the results pertaining to the different lower flow boundary conditions would be useful as lower and upper bounds for a layered soil system seperated by the geometry corresponding to the flow boundary.

CHAPTER 1

INTRODUCTION

Seepage computations play an important role in the design of hydraulic structures, retaining structures and stability of slopes. In case of design of hydraulic structures the specific problems to be dealt with can be classified as:

1. Estimation of quantity of seepage and zone of seepage.
2. Uplift pressure distribution at the base of the structures, and
3. Estimation of exit gradient to safeguard against piping.

It should be noted that in many instances the damage occurring in hydraulic structures can mainly be attributed to the destructive effect of seepage.

Eventhough the basic concepts of theory of fluid flow through porous media have been expounded nearly a

century back, it is only since the beginning of twentieth century the subject has received a scientific treatment. One of the reasons for chequered development of the subject in the early stages has been the heterogeneous nature of the soil at microscopic level. However, now it is well accepted fact that most of the seepage and groundwater flow problems can be analysed on the basis of Darcey's simple linear law proposed in 1856. In addition to the proof of validity of Darcy's law, the developments in hydraulics and hydromechanics have made it possible to derive several theoretical solutions which are applicable to many field situations.

Most of the available theoretical solutions of groundwater flow assume the porous media to be isotropic and homogeneous with respect to coefficient of permeability. However, materials occurring in nature are seldom isotropic and even less frequently homogeneous. While many adequate general methods of solving seepage problems through isotropic and homogeneous porous media have been developed, problems of confined flow under weirs resting on anisotropic and non-homogeneous soils have received little attention due to associated

mathematical difficulties. Flow through anisotropic porous media is generally analysed by first transforming the anisotropic actual flow domain to a fictitious isotropic flow region by a suitable co-ordinate transformation and then solving the problems for the equivalent isotropic system. For the problem of confined flow below hydraulic structures resting on non-homogeneous soils, theoretical solutions are only available for the case of a two layered soils system with both the layers having same thickness. However, in nature the non-homogeneity can occur in the form of localized pockets of impermeable or permeable soil and jointed rock forms having primary and secondary permeabilities. For the forementioned situations no theoretical solutions are available in literature because of the complex flow boundary conditions at the interface of joints.

Review of the available literature reveals that several methods have been used for finding solutions to problems of flow through porous media. Some of the commonly adopted methods are: conformal mapping finite difference method, finite element method, graphical

approach, energy methods and analog methods. Many problems which are difficult to be solved analytically can be relatively easily solved by electrical analog method. It has been found that the accuracy of the results obtained by analog method is good enough for many engineering problems.

As has been stated earlier, the available knowledge regarding the confined flow problems for hydraulic structures resting on anisotropic and non-homogeneous soil is far from complete. With this in view the present investigation is primarily concerned with the study of some cases of two dimensional confined flow under weirs resting on non-homogeneous and anisotropic media, using electrical analog approach. Also the present study primarily confines only to the distribution of uplift pressure at the base of hydraulic structures.

The scheme of presentation in the thesis is as follows:-

A review of the literature in the field of seepage in anisotropic and non-homogeneous porous media and use of electrical analogy relevent to the present study has been presented in Chapter 2.

Chapter 3 deals with the elements of electrical analogy, its usefulness and its limitations for solving problem in groundwater and seepage. Details of the analog field plotter used in the present investigation are also presented.

Chapter 4 deals with the problems of confined flow through the jointed rock mass which is a particular type of non-homogeneity commonly encountered.

For this case it is difficult to arrive at exact solution due to existence of two permeabilities, the primary permeability of the rock mass and secondary permeability of flow through joints. Recently Madhav and Lakshmidhar (1968) presented analog solution for flow below the weir founded on a jointed rock mass. To study the effect of jointing the joint was either considered to be horizontal or vertical and results thus obtained qualitatively brought out the effect of such major joints on the flow behaviour. In the present work analysis is presented for a weir with a vertical sheet pile founded on jointed rock mass. For simplicity the joint is assumed either vertical or horizontal (Fig. 1(a) and 1(b)). As the effect of joint has been simulated by

drawing a strip of silver paint on analog paper, the results are once again qualitative than quantitative. However, in the absence of any available solutions the analysis will serve to bring out the influence of joint position as affecting the uplift pressure. Results are presented in non-dimensional form for various locations of joints, length and disposition of sheet pile.

The effect of flow boundary as affecting the uplift pressure for a weir resting on anisotropic soil is analysed in Chapter 5. In the initial portion of the chapter, review of the transformation technique is presented which transforms the anisotropic region into an equivalent isotropic region. In the later part of the Chapter results of the study bringing out the effect of lower flow boundary are presented. Various cases analysed are:

(i) Confined flow under a weir on anisotropic soil of finite depth with sloping impervious boundary (Fig. 2).

(ii) Confined flow under a weir resting on an anisotropic soil of infinite depth with sloping pervious boundary (Fig. 3).

(iii) Confined flow under a weir on an anisotropic soil of finite depth underlain by a draining layer and with sloping pervious boundary (Fig. 4).

For all the cases considered, the effect of various parameters as affecting the uplift pressure is presented in the form of non-dimensional charts. It should be noted that for various cases considered, no theoretical solutions are available at present and as such the analysis would be useful while dealing with problems encountered in field with such complicated flow boundary conditions. Also as it is extremely difficult to obtain theoretical solutions for a layered soil system which is not parallel, the solutions can be thought of either lower or upper bound to a two layer soil system separated by the flow boundary which is either assumed pervious or impervious.

The last Chapter (6) deals with the general conclusions based on the present study and also includes suggestions for further research.

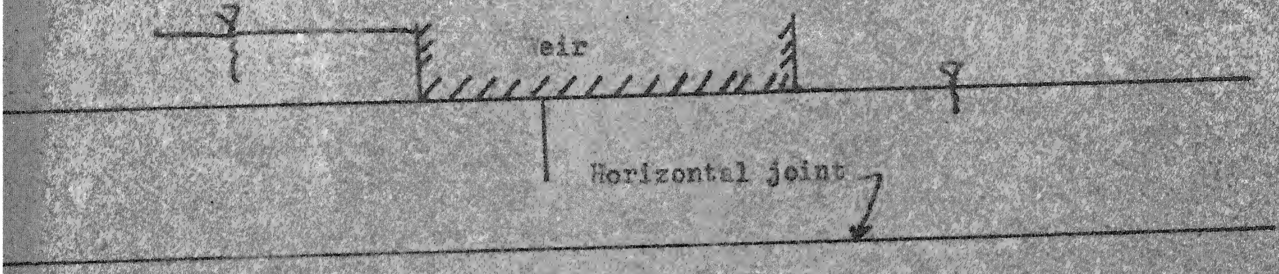


Fig. 1(a) Weir founded on rock foundation with horizontal joint.

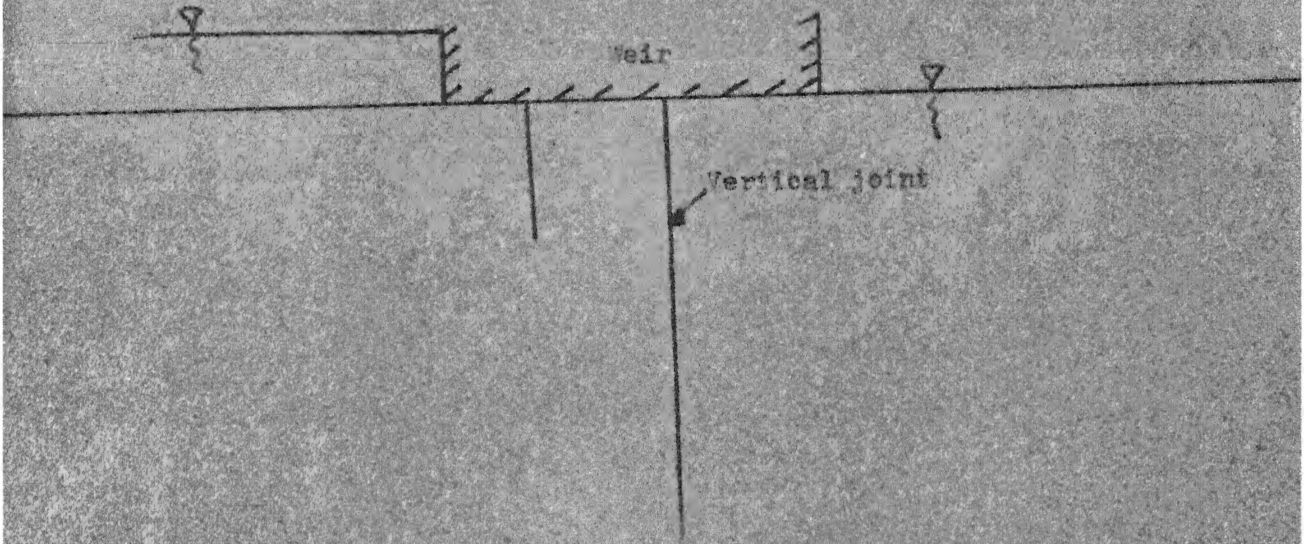


Fig. 1(b) Weir founded on rock foundation with vertical joint

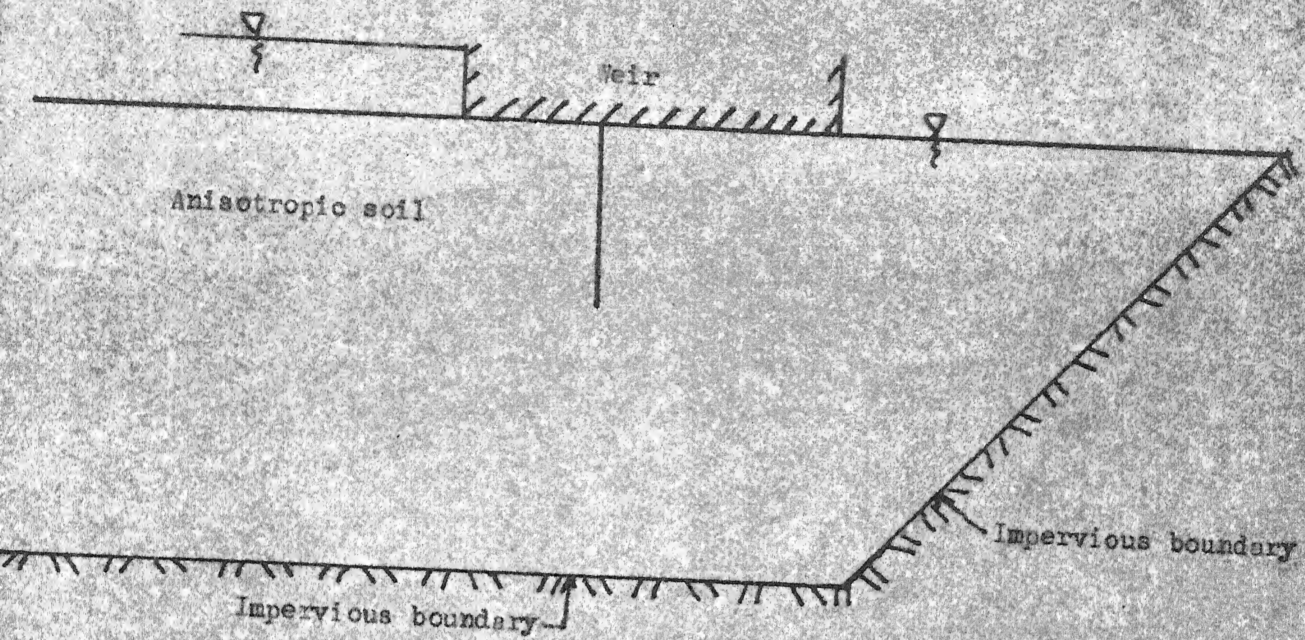


Fig. 2 Weir on anisotropic soil of finite depth with sloping impervious boundary.

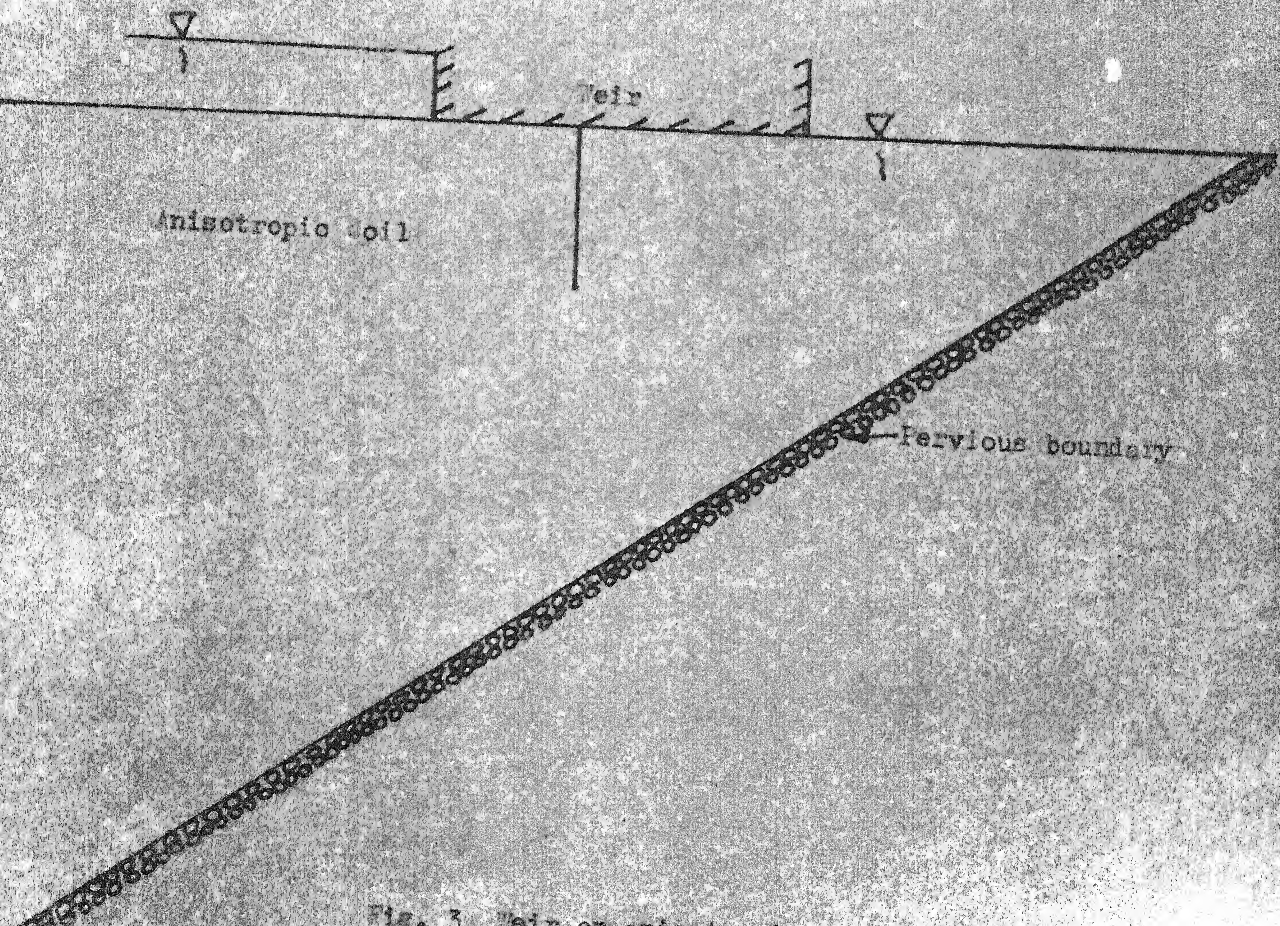


Fig. 3 Weir on anisotropic soil of infinite depth with sloping pervious boundary.

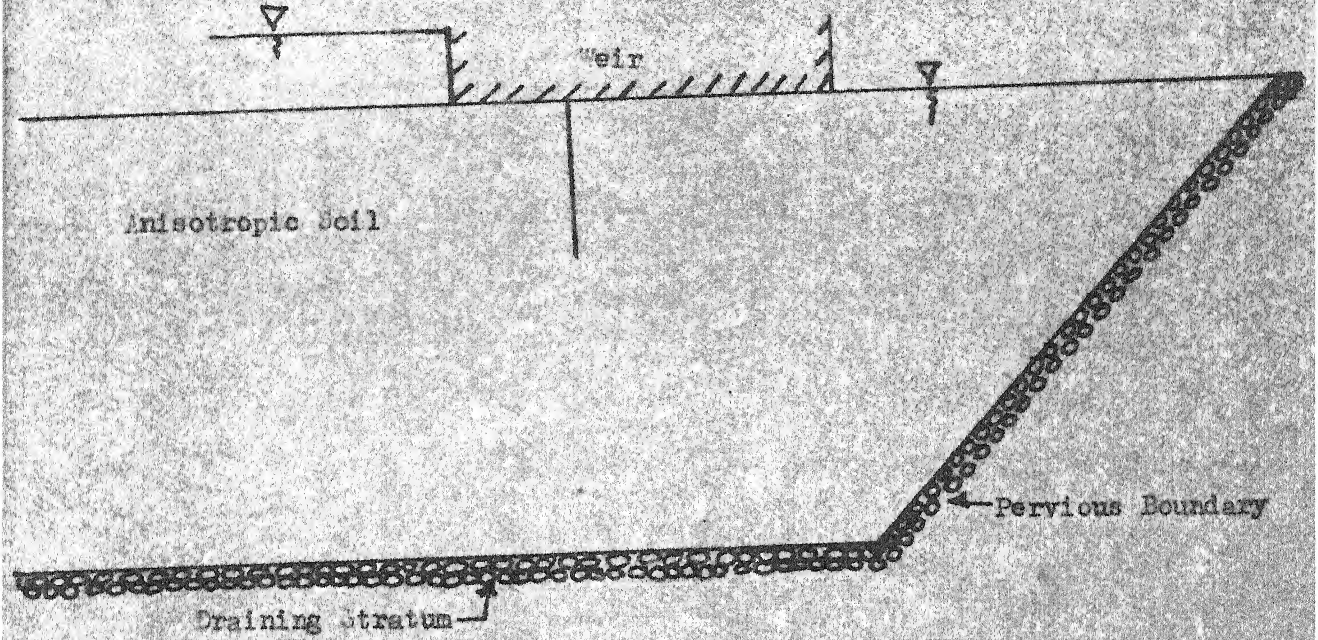


Fig. 4 Weir on an anisotropic soil of finite depth underlain by draining stratum with sloping pervious boundary.

CHAPTER 2

REVIEW OF LITERATURE

2.1 GENERAL:

In this Chapter a brief review of the literature pertaining to the two-dimensional confined flow problems is presented. In the first section, review is presented of the various analytical investigations dealing with the problem of weirs on permeable foundations. More details are given of the recent studies which include the effect of anisotropy and non-homogeneity in the analysis. In the second section of the chapter, review of relevant electrical analog studies is presented.

2.2 THEORETICAL SOLUTIONS:

From the time Darcy's (1856) established the linear relationship between the seepage velocity and hydraulic gradient, many theoretical solutions have been presented for both confined and unconfined problems. Some of the notable contributions which have a direct bearing on the present investigations are: the case of confined flow

under a weir with a vertical sheet pile and resting on porous media of infinite depth (Khosla, etal 1954), confined flow under a weir with a vertical sheet pile and resting on a finite porous layer underlain by either a impervious stratum (Muskat(1937), Pavlovsky (1933)) or a horizontal draining layer (Grinsky 1950) , confined flow under a depressed weir (Pavlovsky 1933) the case of an inclined sheet pile in a semi-infinite horizontal porous medium (Vergin, 1940) and the case of a confined flow under a weir on an anisotropic medium of finite and infinite depth (Reddy and Mishra 1972). Theoretical solutions considering the non-homogeneity of the porous medium are presented by Polubarinova Kochina(1962) for the cases of vertical sheet pile and a weir without sheet pile, resting on a two-layered soil having individual layers of equal thickness.

In all the above cited works solutions have been obtained by means of Schwartz Christoffel transformation technique. The final solutions are obtained in general in terms of hyperbolic functions, or elliptic functions or incomplete and complete beta and gamma functions.

Most of the above theoretical solutions are based on the assumption of isotropic and homogeneous soil medium.

However, in reality, one often encounters situation where the soil is either anisotropic or non-homogeneous or both.

A general theoretical analysis of flow of fluids through anisotropic soils has been presented by Ferrandon (1948), Scheidegger (1957), Polubarinova-Kochina (1962), Harr (1962), Marcus (1962) and more recently by Misra (1972).

In general for taking into account the anisotropic nature of the soil, the actual flow domain is transformed into an equivalent isotropic flow domain by co-ordinate transformation. The details of transformation can be found in the book by Harr (1962), Polbarinova - Kochina (1962), and De-wiest(1965). By making use of the fore-mentioned transformation technique, Virgin (1940) has solved the problem of a vertical sheet pile in an anisotropic medium. Adopting essentially the same approach (Reddy and Misra 1972) have solved the problem of confined flow under a weir on an anisotropic medium of finite and infinite depth.

Also in many instances, the region in which the seepage takes place is essentially of a non-homogeneous character. In such conditions seepage problems become

complicated. For example problems of seepage beneath water retaining structures often require the considerations of layers of soil with different permeabilities which also may be anisotropic. The permeability being a function of void ratio, may decrease with depth within a layer and stratum may be lense like. If the non-homogeneity can be characterised by stratification consisting of sequence of thin homogeneous isotropic layer of different permeability and thickness, the problem can ultimately be converted into a form that is tractable by one of the technique listed in Chapter 1. In general this is not the case and there are fewer methods available for the solutions of irreducible non-homogeneous, anisotropic seepage problems. For simple flow situations Pavlovski and Kamenski have given some approximate methods for reducing a multilayered soil to equivalent homogeneous one.

Flow net sketching can be done in practice, but it is not practicable except where the non-homogeneity is relatively simple. The Hele-shaw model can be used for non-homogeneous isotropic conditions but requires carefully constructed appratus. The most suitable methods in current use for solving more general problems are electrical analog and numerical methods. The electrical

resistance network Scott (1963), Herbert and Ruston (1960) provides a technique of considerable versatility which may also be used for 3 - dimensional problems. A wide variety of problems can also be solved numerically using digital computers. The finite elements method Zienkiewicz (1966) can be effectively used for analysing the problems of flow through anisotropic and non-homogeneous soils.

Flow through jointed rock is also a case of flow through non-homogeneous porous media due to occurrence of two permeabilities of rock mass. Kransnopol'skii (1912) proposed a formula for seepage in jointed rocks, according to which the head loss is proportional to the square of seepage velocity.

A similar formula was derived by Puzyrevskii (1930) for seepage in a stone talus.

Serafim and Campo (1965) calculated the average permeability Kav for the rock mass as:

$$K_{av} = \frac{\gamma_w}{12\,\mu} \left(\frac{e^3}{d} \right)$$

e = width of joint;

d = spacing of joint;

γ_w = unit weight of fluid;

and μ = viscosity of the fluid.

The above equation is useful when one is interested in calculating the quantity of seepage through the rockmass, but estimation of pressure head at different location is not possible with the above equations.

Wittke and Louis (1966) neglect the primary permeability of the rock mass and base their analysis on the geometry of the joints. Flow equations are developed by Louis (1966) for seepage through open or filled joints considering the form and the surface roughness of the joints.

For quantitative study finite element methods (Zienkiewicz (1966)) have been more readily applied to the seepage through rock foundation. A simple numerical procedure has been recently developed by Madhav (1978) to make quantitative study of flow through jointed rock mass.

2.3 ELECTRICAL ANALOG SOLUTIONS:

From analogy between Ohm's Law for condition of electricity and Darcy's Law for flow of water through porous medium; Pavlovsky suggested as early as in 1921,

that an analog method could be employed for the purpose of studying the uplift pressure under dam. Some attempts were made in this direction by Lane & Coworkers (1954) in America but no definite conclusion were arrived at. However, subsequently the analog method was improved and was successfully used by many research workers notable amongst them being Hazra (1935), Wycoff and Reed (1935), Selim (1947) and Vaidhianathan (1955).

For the first time three dimensional electrical analog technique was used by Reltov in 1936 to study the flow pattern under a hydraulic structure built on a heterogeneous soil mass.

Hazra (1935) employed this technique to investigate (i) uplift pressure (ii) exit gradient and (iii) quantity of seepage under dam on sand. Electrolytic tank of size 24 x 26" with conducting media as salt solution was used for conducting experiments. Also the results of his study showed remarkable closeness to the already available theoretical solution, thus proving the usefulness of analog technique.

For the analysis and design of weirs resting on permeable foundations Vaidhinathan et al (1955) used extensively the electrical analogy method. They studied

the uplift pressure distribution for the cases:

(i) A simple impervious floor flush with the surface of the porous strata.

(ii) The same case with sheet pile.

For both cases, theoretical solutions and direct measurement in hydraulic models existed, so the results of the electrical analogy method could be compared with direct observation of pressure and with theoretical solutions. In all the cases the agreement between electrical analog solution, theoretical solutions and observed direct measurements of hydraulic models was reasonably good. Subsequently Vaidhianathan (1955) presented results of exhaustive investigation pertaining to the weirs with number of sheet piles resting on finite and infinite porous stratum. In addition to the above mentioned problems Vaidhinathan (1955) successfully used this method for solving problems relating to depressed weirs, tube wells, coffer dams and earthworks. The appratus employed by Vaidhianathan, for the study consisted of two parts the nodel tank (4'x3' x 3") and the source of alternating current. Two conducting plates of copper each 1.5' long respresenting the surface of the porous strata on the upstream and downstream of the dam

were used. The base of the dam was simulated by 1' long ebonite plate, flush with conductors. The potentiometer / wires served as a potential divider. Luthra and Joglekar (1961) employed electrical analog technique to investigate the distribution of uplift pressure below hydraulic structure for the situation when the sub-soil though permeable, is in stratified layers. They considered the following aspects of stratification:-

(a) Cases where a number of layers of different permeability exist in subsoil.

(b) Cases where an impervious stratum exist under a pervious sub-soil medium.

(c) Cases of an impervious layer of irregular shape and localised pockets of impervious soil.

Meehan, R.L. and Morgenstern, N.R. (1968) employed th technique for getting approximate solutions of seepage problems for following cases:-

(i) Sheet pile cutoff in homogeneous isotropic soil.

(ii) Sheet pile cutoff in layered system, and

(iii) Earthdam on anisotropic layered foundation.

They employed a conducting resistance networks which was prepared by drawing lines of graphite ink on a sheet which conducts electricity when dried. Permeability was varied by increasing or decreasing the spacing of lines.

Stefen, H. and Coworker (1964, 1970) solved the problem of two dimensional seepage under a flat-bottomed structure with a sheet pile cutoff wall at upstream end resting on two horizontal layers of different permeabilities and of equal thickness. Conducting paper was used to represent flow field. Since the permeability is proportional to the number of layers, it is possible to adjust permeability locally by using several layers. In order to vary permeability ratio which ranged from 0.25 to 8.00, several sheets of paper were glued together with amylacetate. The experimental technique was verified against the analytical solution for seepage flow under impervious structure without a cutoff wall, as given by Harr (1962).

For the analysis of confined flow below a flat bottoned weir resting on jointed rockmass Madhav and Lakshmidhar (1968) used the electrical analog method.

Analog field plotter was employed in the study. Conducting graphite (Teledeltos) paper was used for drawing equipotential and flow lines. Primary permeability of the rock mass was simulated by electrically conducting graphite (teledeltos) paper while secondary permeability by a thick line of silver paint on the paper.

Raza (1971) carried out investigation to evaluate the distribution of uplift pressure below a dam and to determine the effect of upstream sheet piling and presence of localised non-homogeneities on the uplift pressure. He also studied the effect of size, shape and number of discontinuties present in the region below a dam as affecting the uplift pressure.

Punmia and Khullar (1972) presented results of analog study dealing with uplift pressure below apron founded on pervious medium of finite depth with downstream cutoff under scour conditions.

It can be seen from the above brief review that the electrical analog method is one of the varsatile techniques which can be employed for solving problems in the area of seepage and ground water. With this in view in the present investigation electrical

analog technique is used to solve some seepage problems taking into account anisotropy and non-homogeneity for which neither analytical nor experimental solutions are available so far.

CHAPTER 3

ELECTRICAL ANALOGY

3.1 GENERAL:

In many branches of engineering and physics, Laplace's equation is encountered as the field equation. Among these areas are : the steady state flow of groundwater, electricity and heat. The basis of analogy becomes obvious when one considers the governing laws in various areas, that is; Darcy's law of steady state flow; Ohm's law for current conduction and Fourier's law for heat conduction. Theoretically, any of the above analogies can be used for analysing the problem of confined flow. However, building up of a thermal analog model to simulate soil water flow situation is quite a formidable task since it is difficult to form boundaries under the requirement of thermal insulation along the flow boundary and the maintainance of constant temperature along the potential boundaries (Scott, 1963). Because of this, thermal analogy is not used in practice. However, one can conveniently use electrical analogy to solve the problem of steady

state flow through porous medium. Also the electrical analog has added advantage of simplicity of experimentation and the solutions can be obtained with ease and rapidity. The correspondence between the steady state flow through porous media and the steady flow of electric current is presented in tabular form below:

<u>Steady State Seepage</u>	<u>Electric Current</u>
Darcy's law for water seepage	Ohm's law for conduction of electricity
$Q = (KAh/l)$	$I = (CA'V/l')$
1. Q = quantity of discharge	I = rate of flow of electricity
2. K = Coefficient of permeability	C = Current Conductivity Coefficient.
3. A = Cross Sectional Area	A' = Cross-sectional Area
4. h = Pressure head	V = Electrical Potential (Voltage)
5. l = Length of Seepage path	l' = Length of path of electric current

Field Equations:

6. $\nabla^2 h = 0$ $\nabla^2 V = 0$

7. Impervious boundary Insulated boundary

$$\frac{\partial h}{\partial n} = 0 \qquad \frac{\partial V}{\partial n} = 0$$

In case of heat flow through conductors, or that of electricity, an analytical treatment is possible. This is because the conductors in these two cases are generally metals or crystals and they have regular isotropic or anisotropic structure. In contrast, the geometrical boundaries surrounding the intersticial water in sand or caly, always differ and do not repeat in any definite arrangement. Also, the nature of packing changes the transmission i.e. changes the conductivity. Thus, the analogy, therefore failsin the regions of molecular or microscopic behaviour of flow of water through porous media. It can only be viewed as applicable to macroscopic flow (Vaidhinathan 1955).

3.2 ELECTRICAL ANALOG MODELS:

For solving the problems of confined flow through porous media both continuous and lumped parameter analog are employed. The details of some of the commonly used techniques are given below:

(1) <u>Continuous Models:</u>

(a) Electrolyte Trough Method:

This has been used quite extensively to solve two or three dimensional seepage problems. The electrolyte or trough consists of a layer of current conducting electrolyte contained within a suitable insulating trough whose size and shape are regulated to conform with the outlines or boundaries of the problem to be studied. Current is fed into the electrolyte through suitably immersed electrode, and equipotentials lines are traced with a detecting instrument. A schematic diagram of a typical electrolyte trough model is shown in Fig. 3.1.

(b) Analog Field Plotter -

It consists of a thin sheet of electrical conducting paper (0.004" thick) in which an electrical current flow pattern is set up by means of suitably

attached and energized electrodes. The resultant potential-drop established by this current pattern is detected and marked directly on the paper by means of a searching sytlus in conjunction with a high sensitivity detecting instrument. Since the flow of current is in two dimension hence this method is only applicable to the analysis of two dimensional seepage problems.

(2) Lumped Parameter Model:- Fig. 3.2(a) represents a section of soil medium through which steady state seepage is taking place. The actual flow domain has been discretized in finite number of points as shown in Fig. 3.2(b) for developing an equivalent analog. As can be seen from Fig. 3.2(b) in the corresponding analog the continuous nature of the medium is approximated by a number of discrete resistances joining the points a, b, c . . . etc., corresponding to A, B, C . . . . etc. in actual flow domain. Thus the continuous conductance AB, BC, . . . etc. are replaced by lumped resistors ab, bc . . . etc., the analog electrical conductors being related to the hydraulic conductance by a fixed scale factor. Thus, in lumped parameter model a permanent board set with connectors, usually in square array

at a suitable spacing is used to solve the confined flow problems. Detailed account of lumped parameter analog can be found in the book by Scott (1963).

In comparison to analog field plotter electrolyte trough method has the disadvantage that it requires rather elaborate and laborious initial setup. In addition the technique of conducting experiments may involve an appreciable amount of polarization and phase shift in certain cases, and has the inherent difficulty of using a spillable electrolyte whose electrical properties are sometimes difficult to regulate and maintain uniformly constant over a period of time.

In the case of lumped parameter analog model, the network can be extended without any difficulty to radial, two dimensional or even three dimensional problems. However, in case of continuous analog employing dry electrically conducting paper the analysis is possible only for two dimensional problem. Also inclusion of non-homogeneity is relatively easily accomplished in case of lumped parameter analog when compared to continuous analog. Transient

hydraulic flow can also be simulated in a lumped parameter analog by connecting an electrical capacitor to each node point of the analogous electrical net.

In the present investigation as the problems of confined flow to be investigated could be easily analysed on a continuous model, using dry electrically conducting paper, the same has been used for various studies. The details of analog field plotter used are given in following section.

3.3 ANALOG FIELD PLOTTER:

A schematic diagram of the analog plotter is shown in Fig. 2.3. Plate 2.1 shows the photograph of the complete setup. The various components of the analog plotter are:

(A) Conducting Paper Sheet:

The sheet of conducting paper in which the electric current analogy field pattern is established consists of a piece of type 'L' teledeltos paper approximately 0.004" thick. The conducting paper having resistance of approximately 4000 Ω/sq. has been found very well suited to the use in the field plotter. The ratio of the paper resistance (4000 Ω/sq.) to silver painted electrodes areas (1 to 4 Ω/sq) is

adequately high to permit the conducting of the element area to be considered as being infinite in most of cases. The average paper resistance uniformity of $\pm 3\%$ to $\pm 8\%$ ~~is~~ is believed to be adequate for ~~by far~~ the majority of field plotting problems. This nonuniformity property in resistance appears to be a permanent property introduced in the processing of paper.

(B) Base Board:

The base board used as a plotting base may consist of an ordinary 23" x 31" drawing board of a $\frac{3}{4}$" thick board of insulating material, preferably of plywood, which has a smooth flat surface on which the conducting sheet is mounted.

(C) Instrument Null Detector:

It serves to indicate the location of the points of zero potential which serves to mark a particular equipotential line being traced. The voltage level of the particular equi-potential being traced, is determined by the particular slider position chosen on the voltage-dividing potentiometer, the voltage level being adjustable in small

increaments anywhere between 0 and 100 of the voltage applied to the plotter.

(D) <u>Searching and Marking Stylus:</u>

The searching probe serves both to explore and locate the zero potential balance points comprising the equipotential lines and as a marking stylus whose point is used to make small indentations or perforation in plotting surface for recording the equipotentials.

The stylus consists of a moderately sharp pointed brass rod with an insulating sleeve, about 5" long. It is connected to the terminal marked P, which is the slider circuit of the instrument voltage divider.

(E) <u>Voltage Divider:</u>

The type of voltage-divider unit multiturn potentiometer has been found to be very satisfactory for use with field plotter, having the marked advantage of high precision coupled with very fine incremental adjustment of the slider.

(F) <u>Electrode and Boundary Control Material & Method of Forming Silver Painted Electrode:</u>

Among the various miscellaneous and supplies

which are used at various times with the field plotter in setting up problems are the following:

(1) Silver conducting paint

(2) Acetone toluol for reducing silver paint to proper consistency.

Since it is essential that the electrode areas be generally of very high conductivity, copper wires have been attached to the left and right-hand electrode strips inorder to insure that these strips are low enough in resistance to represent infinite equivalent conductivity as far as plotting circuit is concerned.

Method of forming silver paint - Low resistance electrode areas may be conveniently established on conducting paper sheet by means of silver conducting paint. Silver conducting paste, which comes with about the consistency of molasses, is thinned from 1 to 3 parts by volume of a thinner solvent such as acetone.

In special cases where exceptionally low-resistance electrode areas may be required a second or third coating of silver paint may be applied after the first coat has dried or both sides of paper may be painted

over the desired electrode areas. It usually takes 24 hrs to get the paint dried.

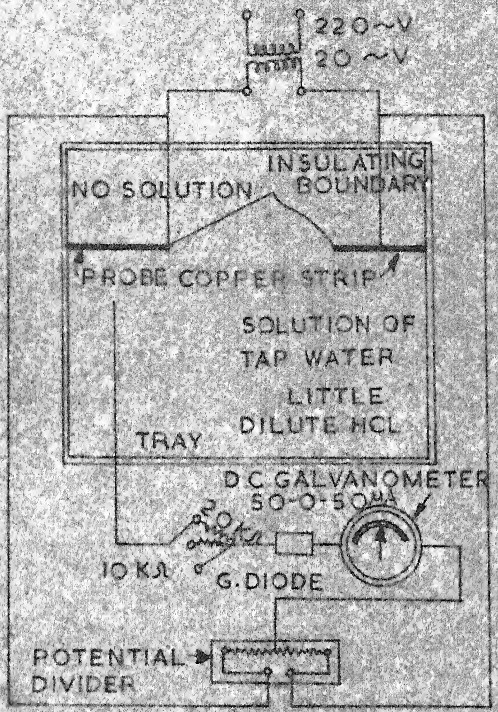


Fig. 3·1 Circuit diagram of electrical analogy tray

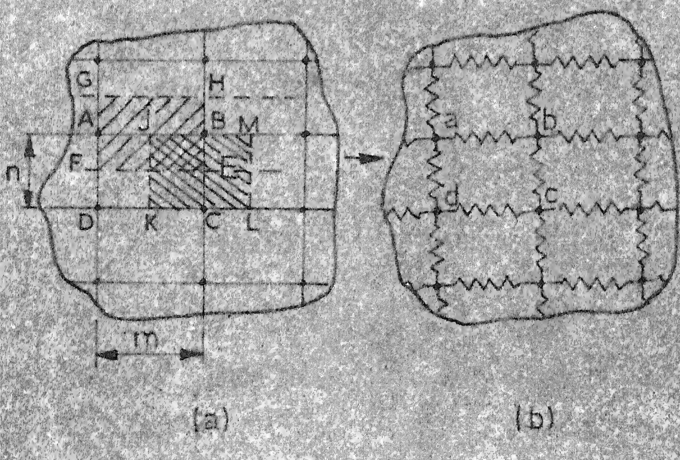


Fig 3·2 Equivalent resistor network (a) physical region (b) resistor model

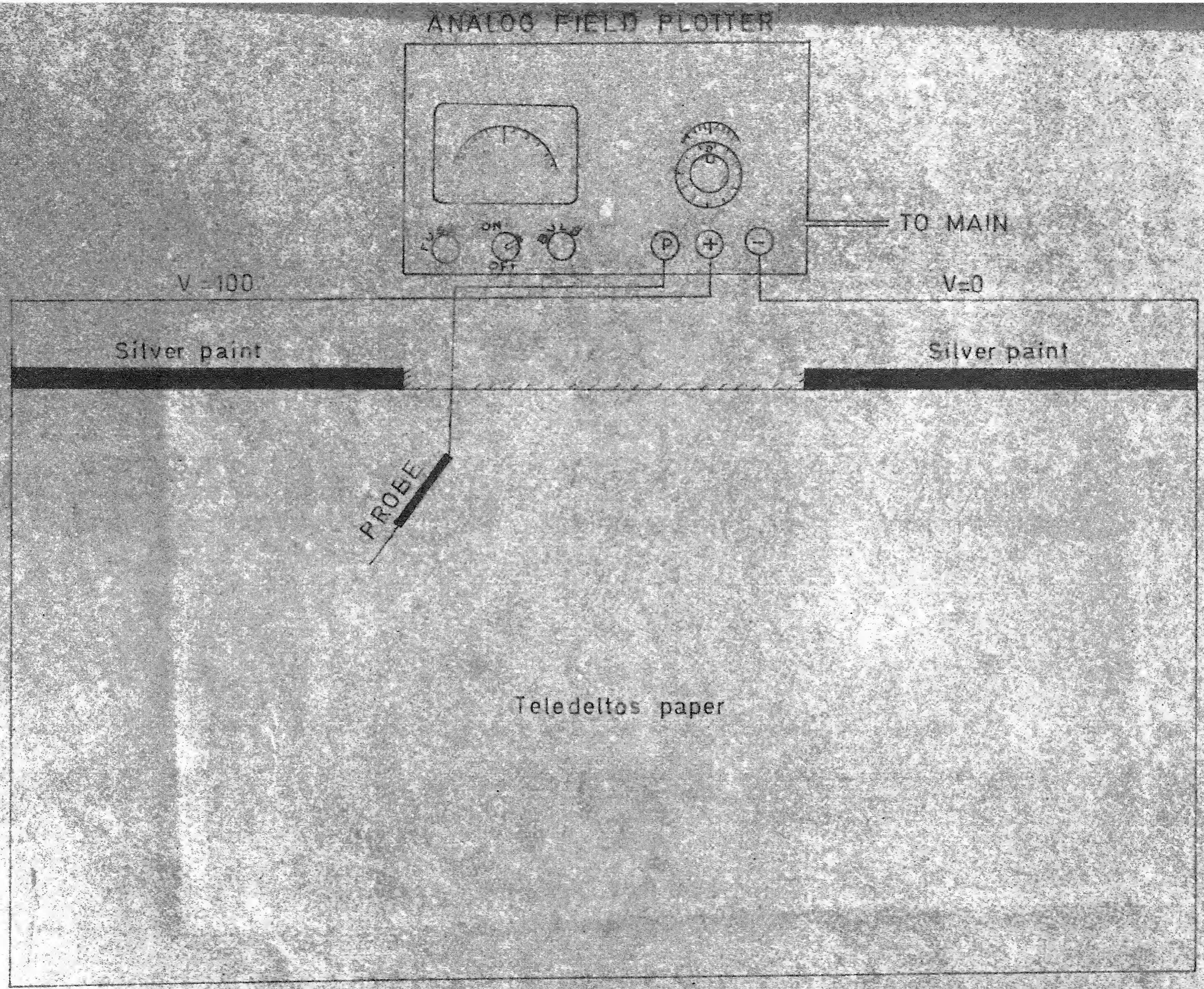


FIG. 3·3 EXPERIMENTAL SET UP FOR TWO DIMENSIONAL CONFINED FLOW PROBLEMS

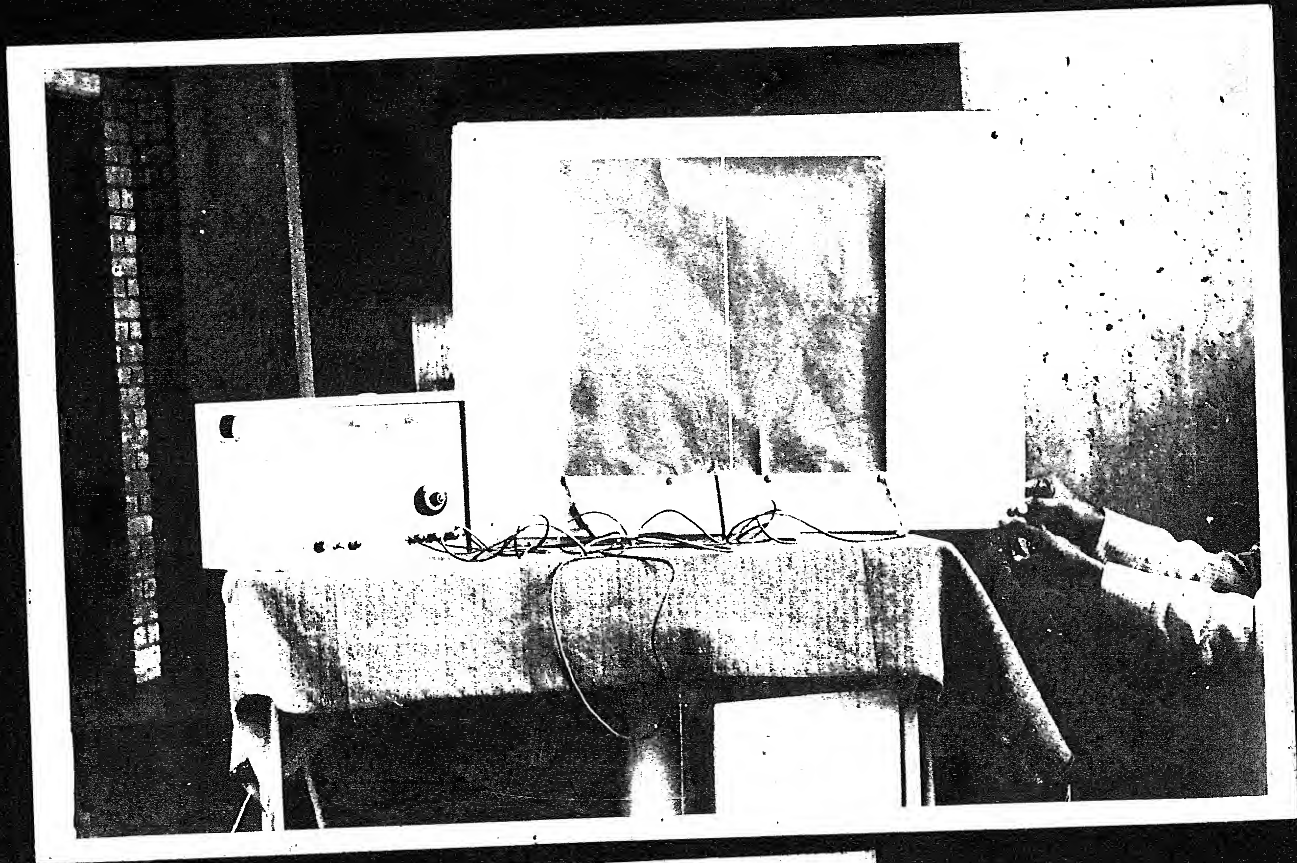


PLATE 3.1

CHAPTER 4

CONFINED FLOW UNDER A WEIR RESTING ON A JOINTED ROCK MASS

4.1 INTRODUCTION:

In many instances hydraulic structures are founded on rock-mass which by definition consists of rock material and planes of weakness due to geological separations. For analysing the flow under such hydraulic structures, it is essential to know characteristics of fluid flow through jointed rock mass. It is obvious that because of the presence of joints, either major or minor, the flow domain can no more be idealised as homogeneous. One of the ways in which the non-homogeneity can be taken into account is by characterising primary permeability for rock material and secondary permeability for joints. By its inherent nature the former is small while the secondary permeability by comparison a much larger parameter. Based on this approach, recently Serafin and Campo (1965) and Wittke and Louis (1966) have presented basic approaches to the analysis of flow through jointed rock mass. Recently in 1968, Madhav and Lakshmidhar developed electrical analog method to study

(i) primary permeability for the rock material and
(ii) Secondary permeability for joints are considered in the analysis. As the end result of any such analysis would considerably depend on the values of these permeabilities, in the following paragraphs further details regarding these parameters are presented.

The primary permeability of rock material bounded by either major or minor joints is due to the presence of voids in rock matrix. For finding the primary permeability in the laboratory, experiments are conducted on cores from rock material either using gas or water as the fluid, (Stagg and Zienkewicz). In general the primary permeability of rock mass is of the order of 10^{-11} to 10^{-9} cm/sec. (Serafin and Campo, 1965). Also the primary permeability is anisotropic in nature being more in horizontal direction compared to the vertical direction.

Terzaghi (1962) defines the secondary permeability as that resulting from the flow through open and continuous joints in the rockmass. It is obvious that by very definition the secondary permeability of a rock mass will be dependent on the opening and spacing of joints. In case of clay filled joints, the permeability

will be a reduced one than the distance between two joint surfaces. For a rock mass with a system of parallel joints of width e, separated by a distance, d, the equivalent permeability (K) of a porous body is given by Serafin and Campo (1965) as

$$K_p = \frac{\gamma_w}{12\mu} \frac{e^3}{d} \tag{4.1}$$

where μ and γ_w are coefficient of viscosity and unit weight of the water. Since the above equation is independent of the geometry of the joint pattern, it would be useful only for calculating the quantity of seepage through rock mass and not for estimating the pressure head at different locations.

The coefficient of secondary permeability can be determined by a "Joint permeability" test and expression for secondary permeability (Ks) can be written as: (Serafin and Campo, 1965)

$$K_s = \frac{e^2 \gamma_w}{12\mu} \tag{4.2}$$

Even for small values of e, the value of Ks worksout to be much larger than the primary permeability (Kp). Depending on the type of rock and the extent of jointing, the ratio of K/kp can be of the order of 10 to 100 or even more.

From the above discussion, it can be concluded that the rockmass can be idealised as a medium with a smaller primary permeability but divided by sets of joints having a high secondary permeability. For the study of confined seepage through such a medium with a view to determine the uplift pressure, very few methods are available. A complex mathematical method has been developed by Barenbldtt et al (1960) which may be difficult for application to many field problems. Also the rockmass cannot be hypothetically replaced by a series of alternate layers having different permeabilities as the joint thickness is negligible in relation to the rock material. Recently Madhav and Lakshmidhar (1968) modified the electrical analog method to simulate the flow through jointed rock mass. They analysed the problems of a weir resting on an infinite rockmass with either a vertical or horizontal joint.

The conducting teledeltos paper served as rock material having primary permeability while the lines drawn with silver paint served to simulate secondary permeability.

Essentially, the same approach as the above is used in the present study to find out the uplift pressure below a weir with a vertical sheet pile resting on jointed rock mass. Fig. 4.1(a) and 4.1(b) show the schematic diagram of the problem under consideration. The different parameters varied during the study are also indicated in Fig. 4.1(a) and 4.1(b).

For presenting the results in non-dimensional form following dimensionless parameters are defined:

$$\lambda = (d/b)$$

$$\eta = (l/b)$$

$$\alpha = (b/s)$$

$$\mu = (b_1/b)$$

where

d = distance of horizontal joint from base of the weir.

b = base width of the weir.

l = distance of vertical joint from 'O' (Fig. 4.1(a) and (b).

b_1 = distance of sheet pile from 'O'

For various values of λ, η, α and μ the uplift pressure distribution at the base of the weir was obtained so also the equipotential lines in the flow domain. Tables 4.1(a) and 4.1(b) summarize the values considered for parameters η, α, μ and λ. As can be seen from tables, the relevent parameters have been varied so as to cover many field situations.

It should be noted that the analysis is confined only either to a vertical or horizontal joint. However the method being general enough can be easily extended to other configurations of single or multiple joints. The details of experimentation are presented in next section.

4.3 DETAILS OF EXPERIMENTS:

The analysis presented is based on the following assumptions:

(1) The joint is either vertical or horizontal.

(2) The rock mass is homogeneous and isotropic.

(3) Joints have been treated as tabular so that they can be depicted by lines in two dimensional representation.

The primary permeability of the rockmass is simulated by the teledeltos paper. To include secondary permeability of the joints, lines can be drawn on the paper with higher conductivity. This can be accomplished in following three ways:

(1) Lines of uniform width and thickness of silver paint

(2) Thin strips of silver foil attached to the teledeltos paper and

(3) Thin electrically conductive wires fixed along the joint location.

Out of these three methods the first one has been used in present investigation because it offers the best contact between the paper and highly conducting medium. The width of the joint should be negligible as compared to the depth of rockmass. Since it is very difficult to draw one single line of uniform thickness and width, lines of 0.2 cm have been drawn. The ratio of joint width to depth of rockmass is 4.4×10^{-3} which is quite small.

In order to reduce boundary effect, the dimension of the model is choosen as follows:

(i) The ratio of the distance left either on right or left side to the half base width is kept 3.5.

(ii) The ratio of the depth of the rockmass to the maximum depth of sheet pile is 6.5.

As per latest work of Mutukumaram and Kulandaiswamy (1972), so long as the distance left on either sides is 2 to 3 times than half the base with of the weir and the ratio of the depth of rockmass to the depth of sheet pile is more than 4.0, the error introduced by boundary effect would be within 3 % . As such, in the present study the error crept in due to boundary effect can be expected to be within 2 %.

4.4 RESULTS AND DISCUSSION:

The results for horizontal and vertical joints as affecting the confined flow behaviour below a weir with a vertical sheet pile are presented below in two sub-sections seperately.

(i) Horizontal Joint:

Figs. 4.2 and 4.3 show the effect of location of joint as affecting the equipotential lines for a sheet pile

at upstream end of the weir for $\lambda = 0.2$ and 0.6 respectively. In both these figures for the same location of joint, equipotential lines are plotted for various length of sheet pile. As it can be seen from these figures, the presence of the joint considerably changes the pattern of equipotential lines. Also by comparing the equipotential lines for the same value of a in both the figures it is seen that the location of joint i.e. its relative distance from the base of the weir also influences the pressure head distribution below the weir. The effect of joint increases with decreasing depth to it from the base of the weir. This is due to the fact that at shallow depth, the joint meets higher equipotentials and these are shifted towards the downstream side. However, it should be once again noted that, as it is not possible to control the thickness of the silver paint, simulating the secondary permeability absolutely, the results should be viewed qualitative than quantitative.

Figs. 4.4 through 4.7 show the distribution of uplift pressure along the base of the weir for $\lambda = 0.2$, 0.4, and 0.6 respectively for various values of μ and α. It should be noted that in the

above figures the values corresponding to $\mu = 0.5$ are not plotted as these were lying closely between the values presented for $\mu = 0$ and 1.0. Also in Fig. 4.4 the values of α are restricted upto only 5 as the joint is located at $\lambda = 0.2$ and as such the minimum value of α for which results could be obtained would be 5.0 so that the sheet pile does not cut the joint. For $\alpha = \infty$, the case corresponds to that of no sheet pile for which the analysis and results have been presented earlier by Madhav and Lakshmidhar (1968). It is seen from Figs. 4.4 through 4.7 that with increasing length of the sheet pile there is considerable reduction in the uplift pressure for the pile located at the upstream end. However, the trend gets reversed when the pile is located at downstream end. These results are in confirmity with the already available solutions based on the assumption of homogeneous porous media.

Figs. 4.7 through 4.9 show the effect of location of sheet pile on pressure head at D (for definition of point D, please refer inset of Fig. 4.7) for $\lambda = 0.2$, 0.4, and 0.6 and for different values

of α. Similar information for point 'C' which corresponds to the tip of the sheet pile is presented in Figs. 4.10 through 4.11. From symmetry, it can be considered that, pressure head at the tip ofthe sheet pile located at the centre of the weir will be 50 % of head difference irrespective of the length of the pile in case of homogeneous porous media. Such a trend is seen in Figs. 4.11 and 4.12 where in the location of joint is relatively away from the base of the weir. However, the pressure head at the tip of the sheet pile falls below 50 % because of the presence of a joint in the near vicinity of the weir as shown in Fig. 4.10.

Figs. 4.13 and 4.14 show the effect of position of joint on pressure head at D for $H = 0$ and $H = 0.5$ respectively for various values of α . From Fig. 4.13 it can be concluded that with increasing distance of location of horizontal joint from the base of the weir the pressure head at D increases and would eventually tend to value corresponding to that of no joint case. Also it is seen that for a sheet pile located at upstream end the pressure head at D is considerably reduced for smaller values of λ meaning thereby a joint at smaller depth. This is an expected

behaviour as the pressure would decrease because of the presence of draining joint in the vicinity of the weir. However, when the sheet pile is located at the centre, the pressure head at D is not markedly influenced as can be seen from Fig. 4.14.

Fig. 4.15 shows the variation of uplift pressure along the base of the weir for the pile at upstream and for various locations of the joint (i.e. for various values of λ). This Fig. clearly bring out the necessity of considering the presence of rock jointing in the analysis as it is seen that the pressure distribution gets considerably altered. Also as reported earlier by Madhav and Lakshmidhar (1968) for $\alpha = \infty$, it is seen that there is increase in pressure at downstream end because of the presence of a joint in the vicinity of the weir for all values of α .

(ii) VERTICAL JOINT -

Figs. 4.16 and 4.17 show the effect of location of joint as influencing the equipotential lines for a sheet pile at upstream end of the weir for η = 0.2 and 0.8 respectively. In both these Figs. for the same location of the joint, equipotential

lines are shown for various values of α. As indicated in the Figs., equipotential lines are considerably affected by the presence of joint. When joint is located towards the downstream end of the weir the effect of joint on equipotential line decreases in relation to the joint nearer upstream end. By comparing the equipotential lines for same value of α, it is observed that the location of the joint i.e. its relative distance from the upstream end affects the pattern of pressure distribution beneath the base of the weir..

Figs.. 4.18 through Fig. 4.20 show the distribution of uplift pressure at the base of the weir for η = 0.2, 0.4 and 0.8 respectively for various values of μ and α. In the above figures the values corresponding to μ = 0.5 are not plotted as these were lying closely between the values presented for μ = 0 and 1.0. It is noticed from Fig. 4.17 through 4.20 that the trend of the influence of α is same as in the case of horizontal joint.

Fig. 4.21 through 4.23 indicate the effect of location of sheet pile on pressure head at D

(for definition of point D, please refer in set of Fig. 4.7) for η = 0.2, 0.4 and 0.8 and for different values of α (5.0, 2.5 and 1.7). Similar information for point 'C' corresponding to the tip of sheet pile is presented in Figs. 4.24 through 4.26. It is seen from these figures that the pressure at the tip of the sheet pile is not 50 % for pile located at the center as is true from symmetry for a sheet pile in homogeneous porous media. This deviation is because of the presence of joints in the vicinity of the sheet piles.

Figs. 4.27 and 4.28 show the effect of location of joint on pressure head at D for μ = 0 and μ = .5 respectively for various values of α. For the pile at upstream end it is seen that pressure head at D would reach an optimum value for a joint located around η = 0.5 for sheet piles having α = 5 and 2.5. However, for the pile located at the center of the weir, the minimum pressure head is attained for a joint located at η = 0.5. This is obvious because in this case because of pronimity of vertical pervious joint there would be considerable reduction in pressure head at D.

Fig. 4.29 shows the variation of uplift pressure along the base of the weir for μ = 0 and 1.0 for

η = 0.2, 0.4 and 0.8. This Fig. clearly brings out the necessity of considering the presence of rock jointing in the analysis as it is noticed that the pressure distribution gets considerably influenced.

TABLE 4.1(a) HORIZONTAL JOINT

μ = 0.0, 0.5, 1.0

λ = d/b	α = s/b
.2	10, 5
.4	10, 5, 2.5
.6	10, 5, 2.5, 1.7

TABLE 4.1(b) VERTICAL JOINT

μ = 0.0, 0.5, 1.0

η = d/b	α = s/b
.2	5, 2.5, 1.7
.4	5, 2.5, 1.7
.8	5, 2.5, 1.7

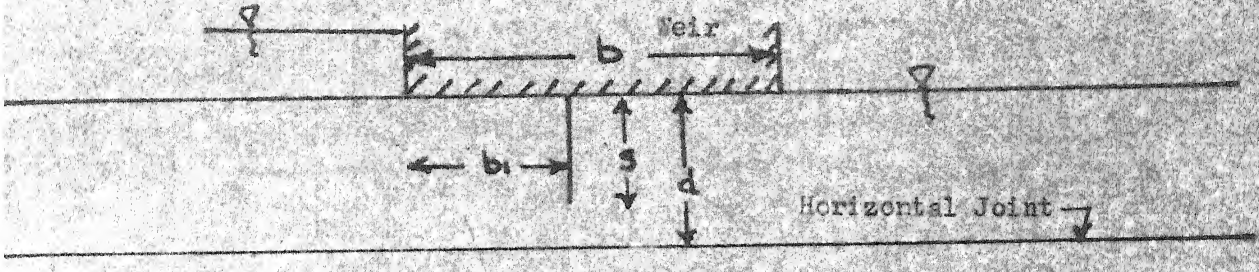


Fig. 4.1(a) Weir resting on rock foundation with a horizontal joint.

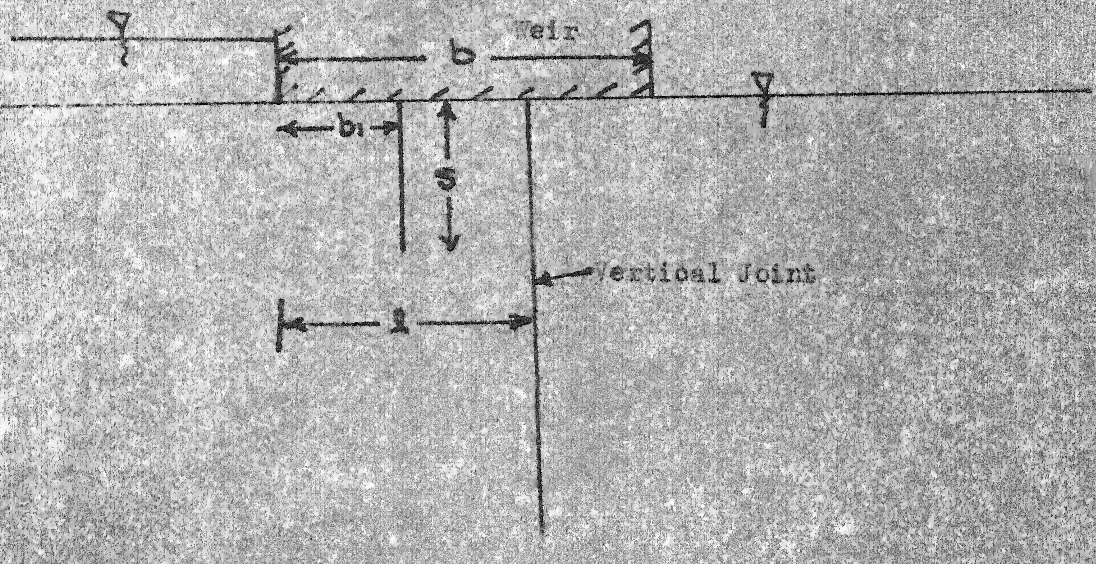


Fig. 4.1(b) Weir resting on rock foundation with a vertical joint.

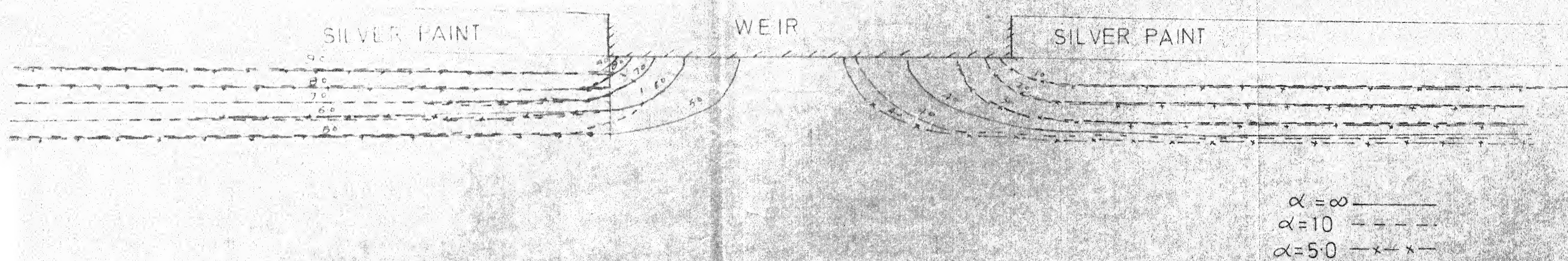


Fig.4.2 Effect of α on equipotential lines for $\lambda = 0.2$, $\mu = 0$

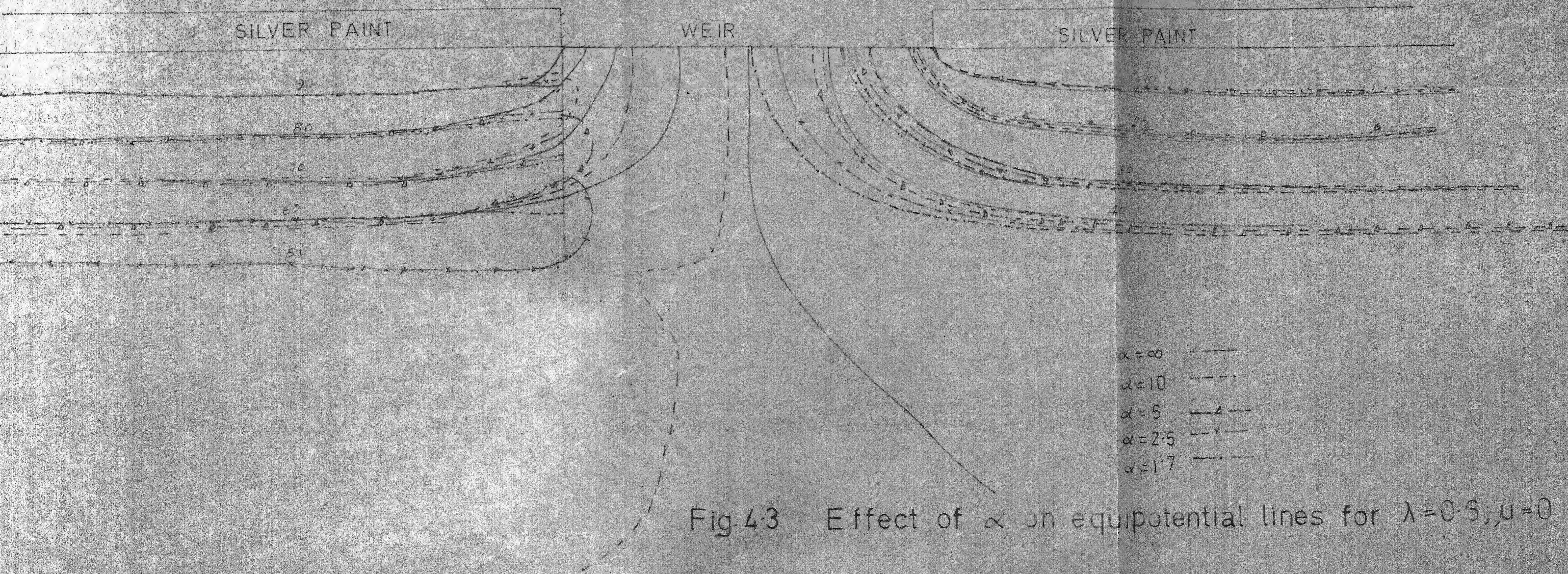


Fig. 4·3 Effect of α on equipotential lines for $\lambda = 0{\cdot}6$, $\mu = 0$

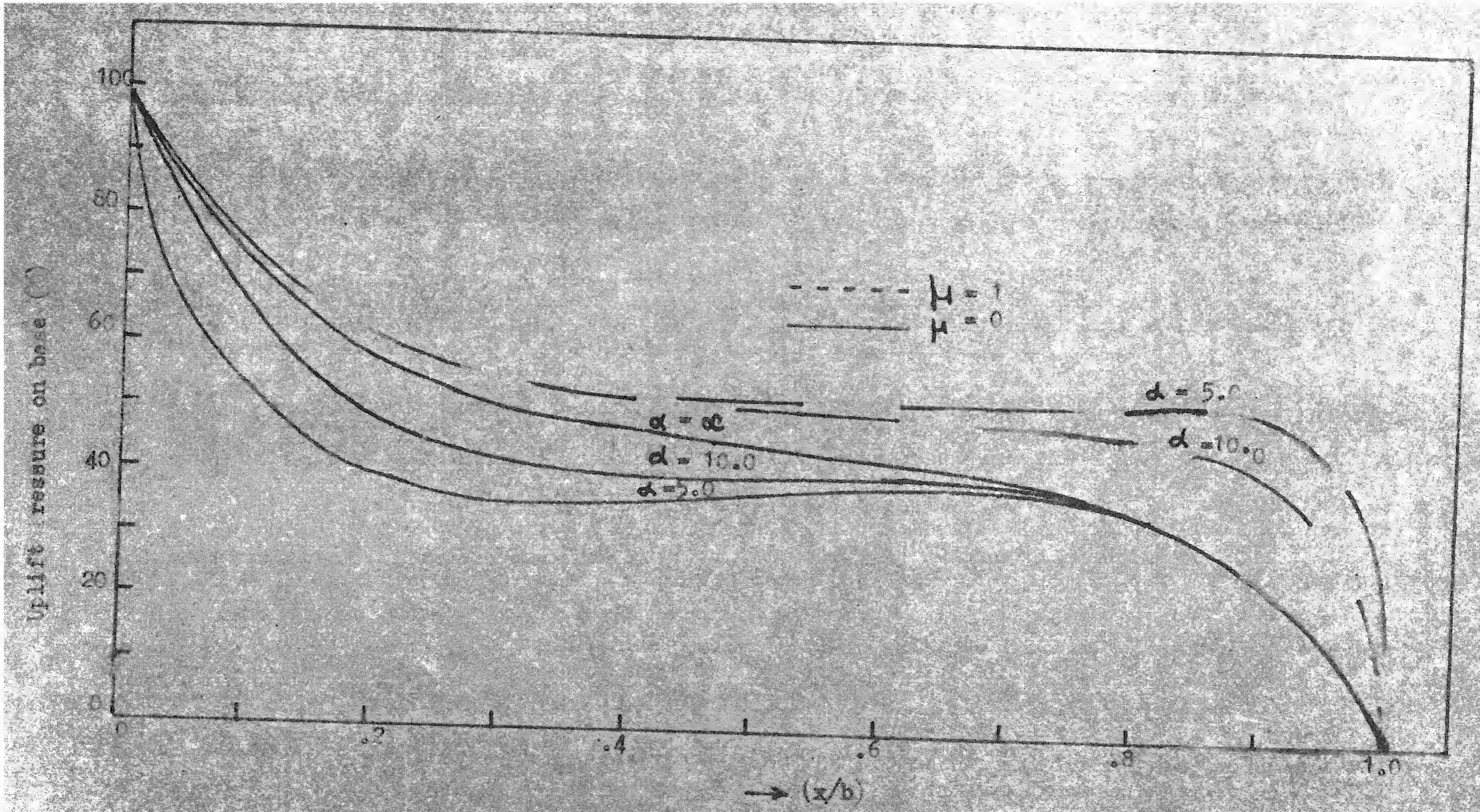


Fig. 4.4 Distribution of Uplift Pressure along the base for $\lambda = 0.2$

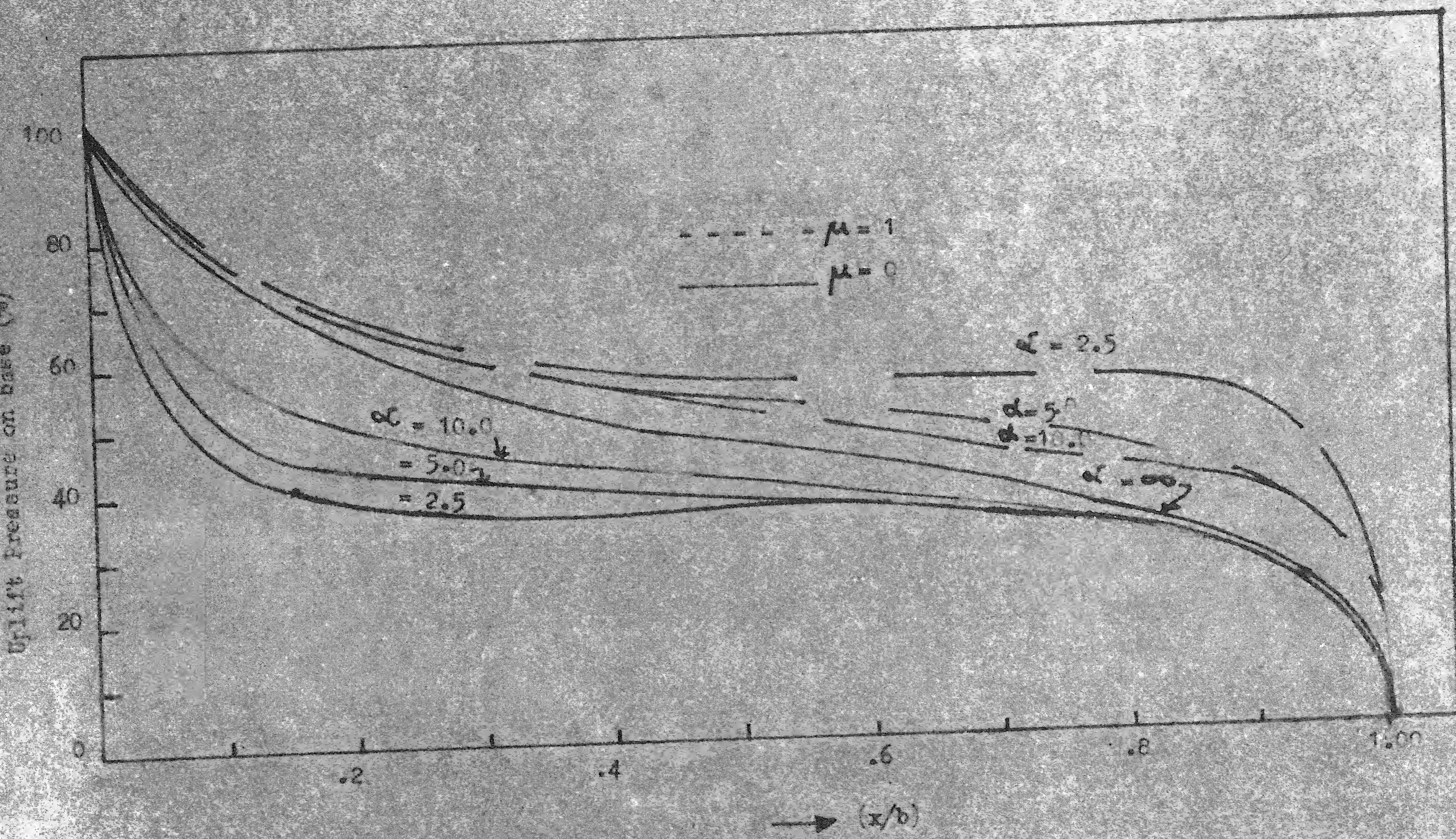


Fig. 4.5 Distribution of Uplift Pressure along the Base of the Weir for $\lambda = 0.4$

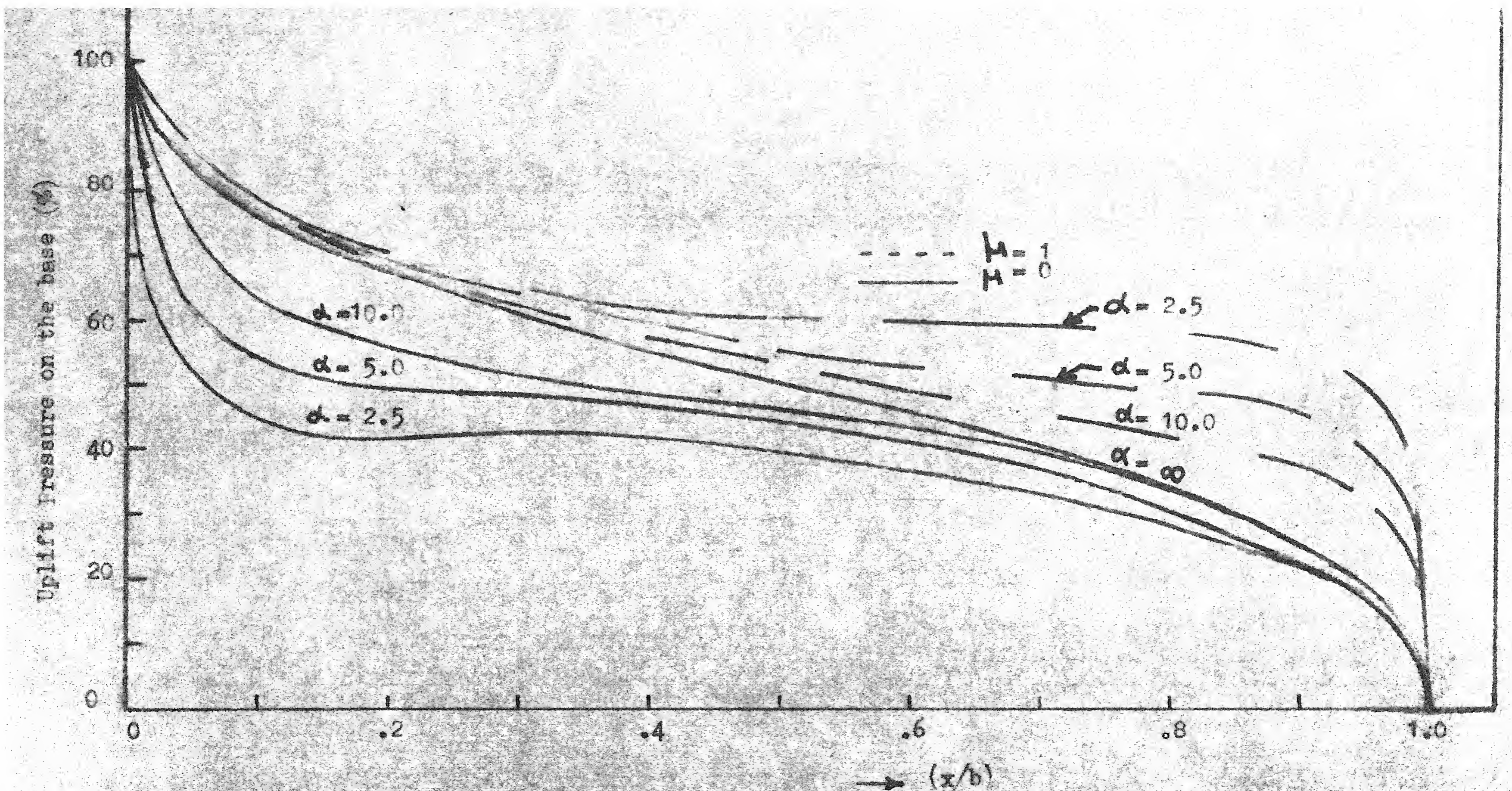


Fig. 4.6 Distribution of Uplift Pressure along the base of the weir for $\lambda = 0.6$

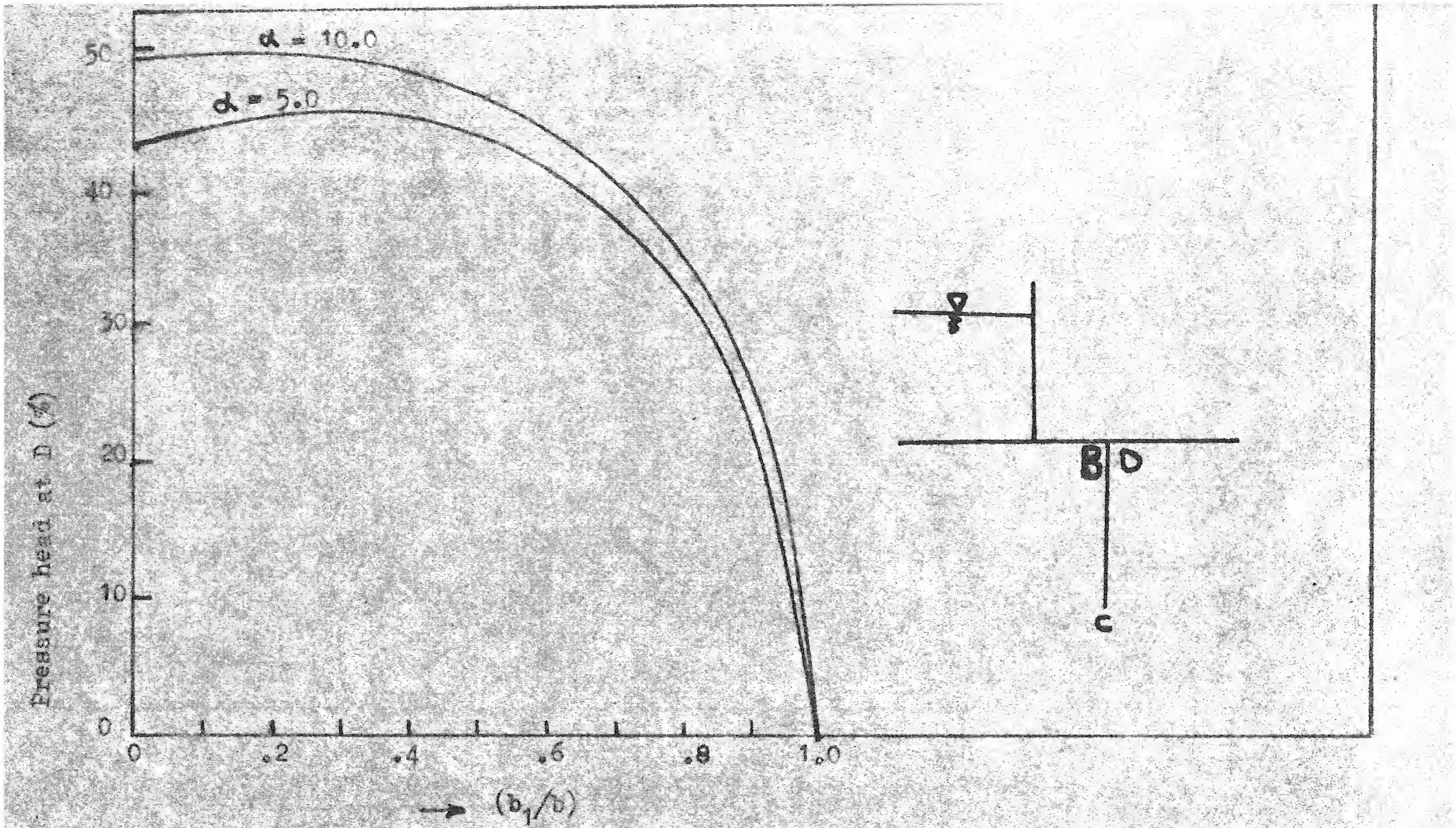


Fig. 4.7 Effect of position of Sheet-pile on pressure head at D for $\lambda = 0.2$

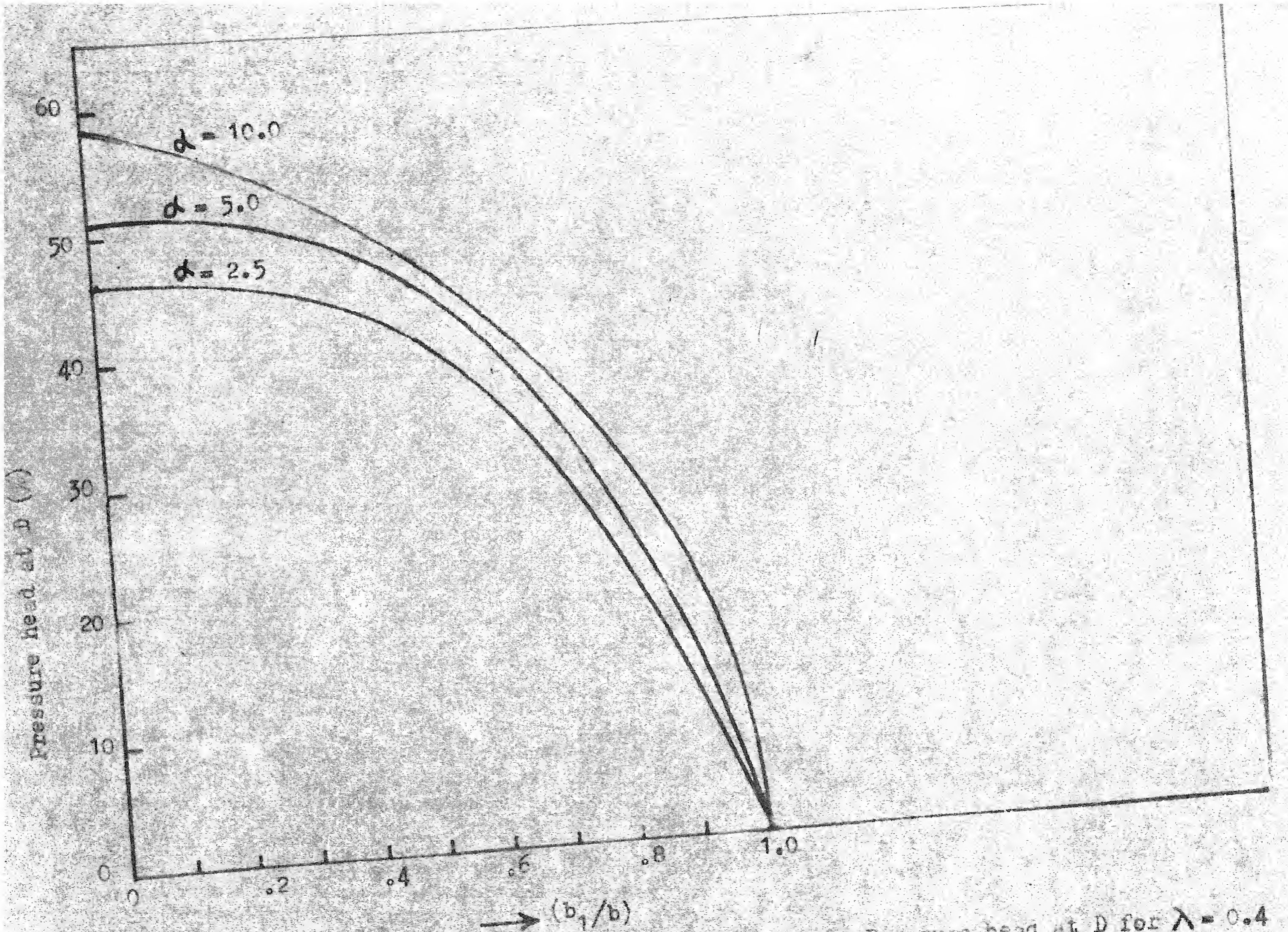
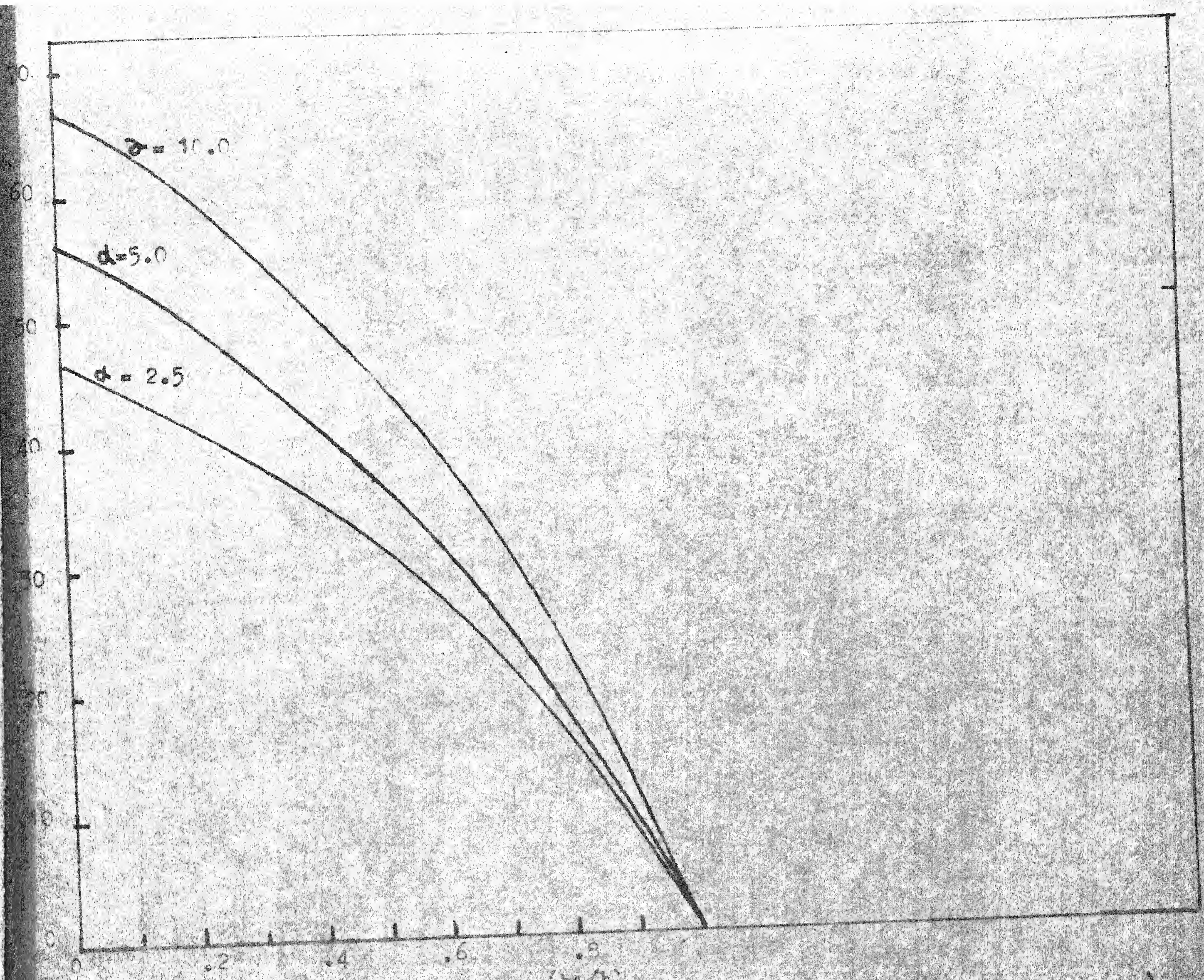


Fig. 4.8 Effect of Position of Sheet Pile on Pressure head at D for λ = 0.4



→ Position of sheet pile on Pressure head at D for λ = 0.6

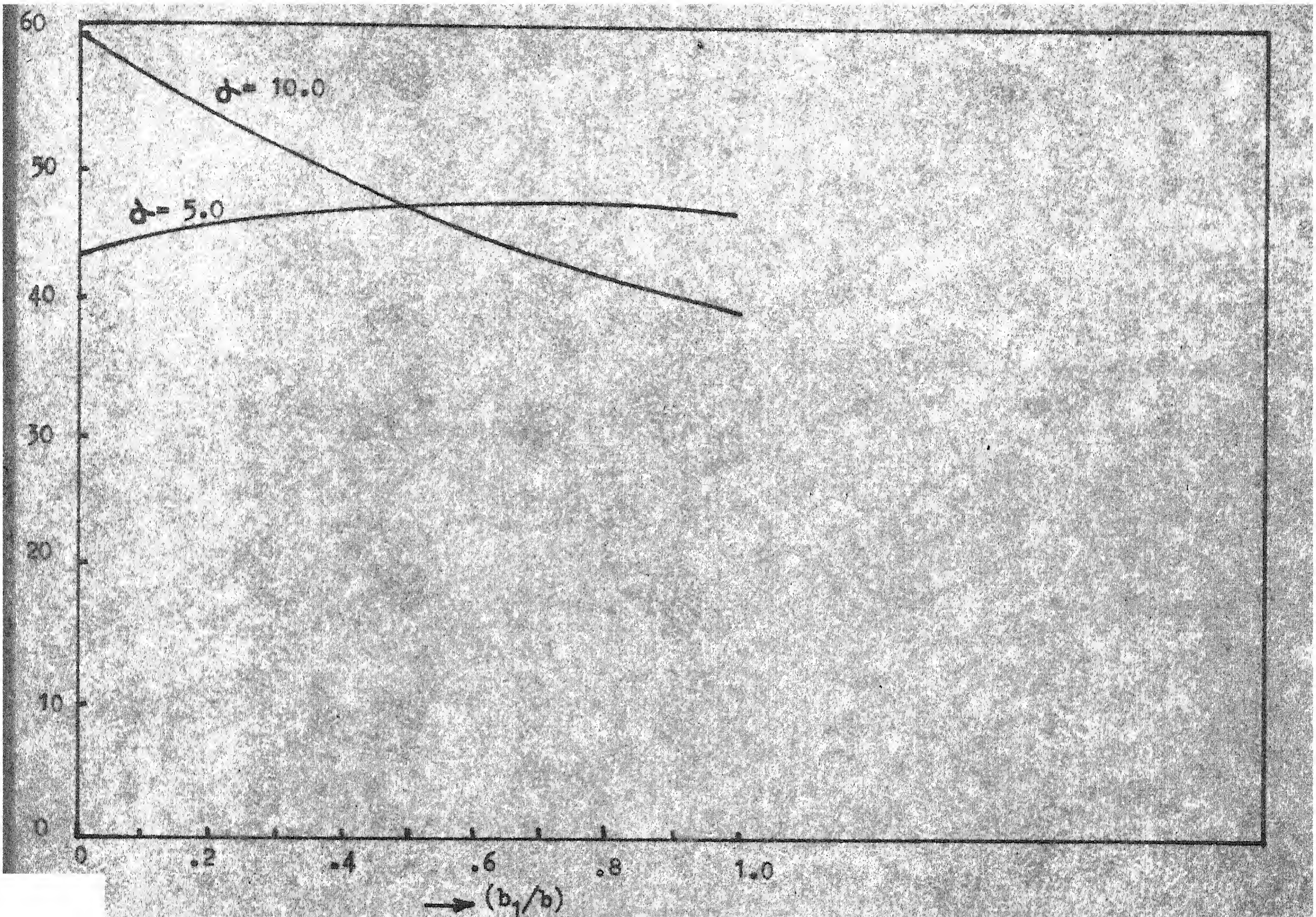


Fig. 4.10 Effect of position of Sheet File on Pressure head at C for $\lambda = 0$.

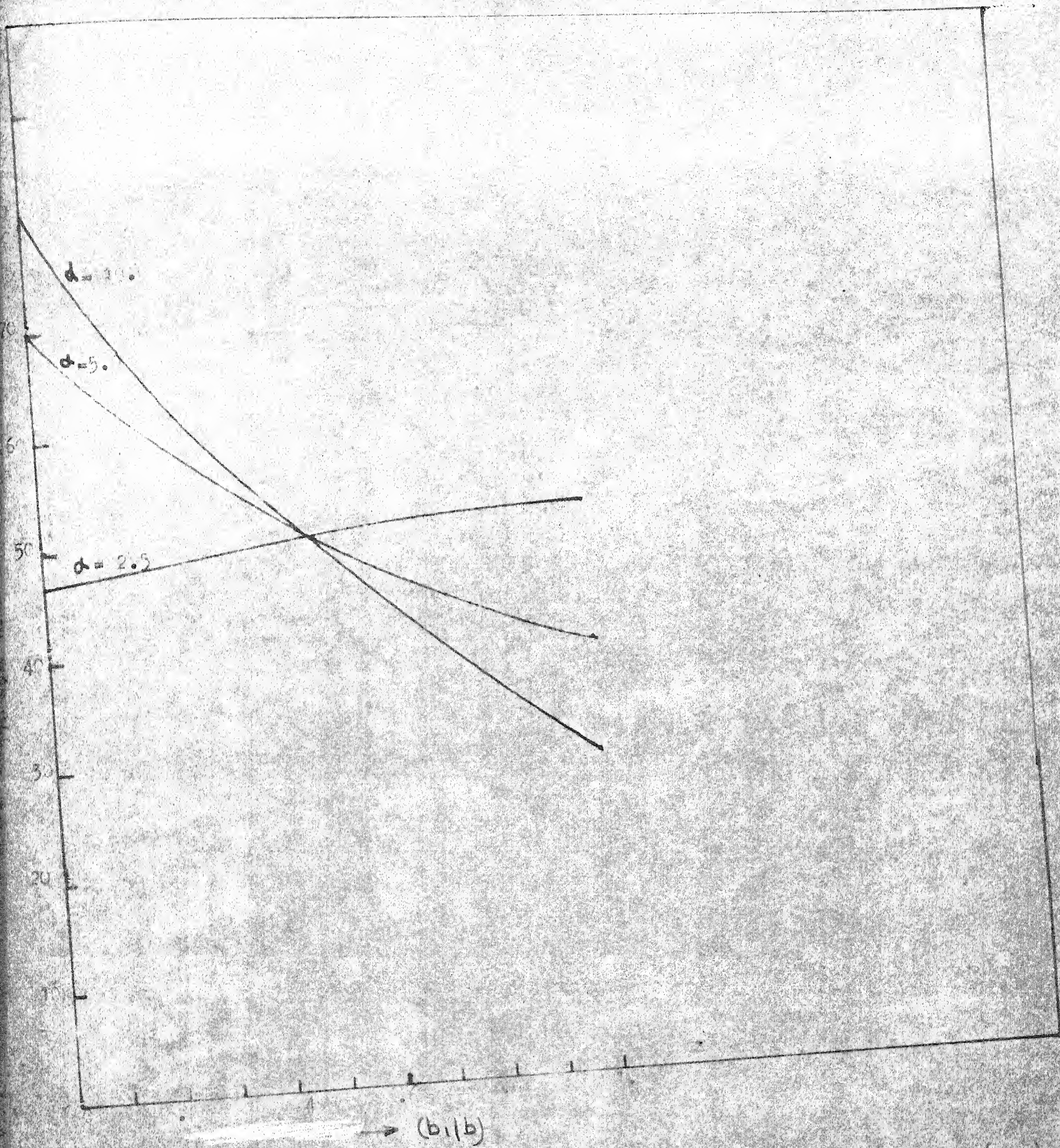


Fig. 4.12   Effect of position of sheet pile on pressure head at c   for λ = 0.6

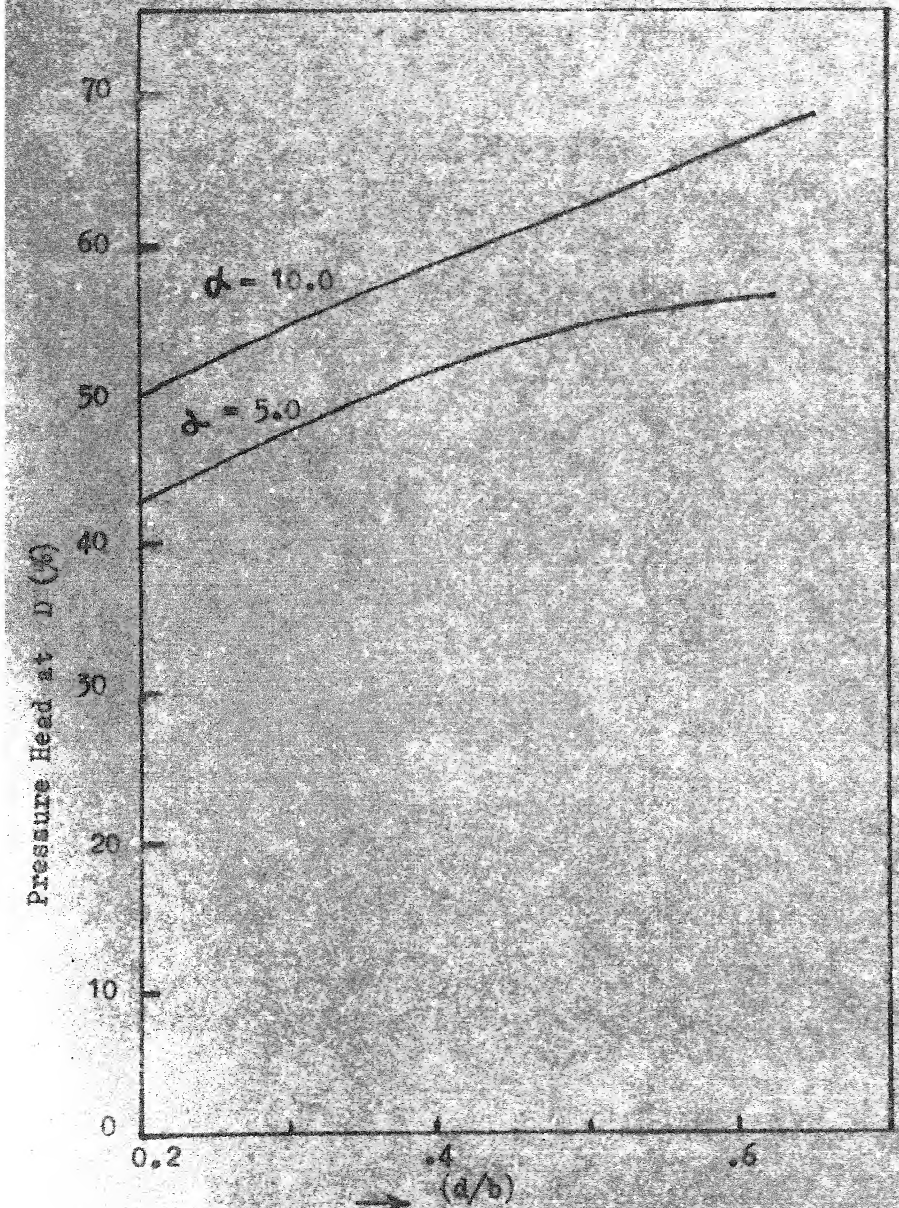


Fig. 4.13 Effect of Position of Joint on Pressure Head at D for $\mu = 0$

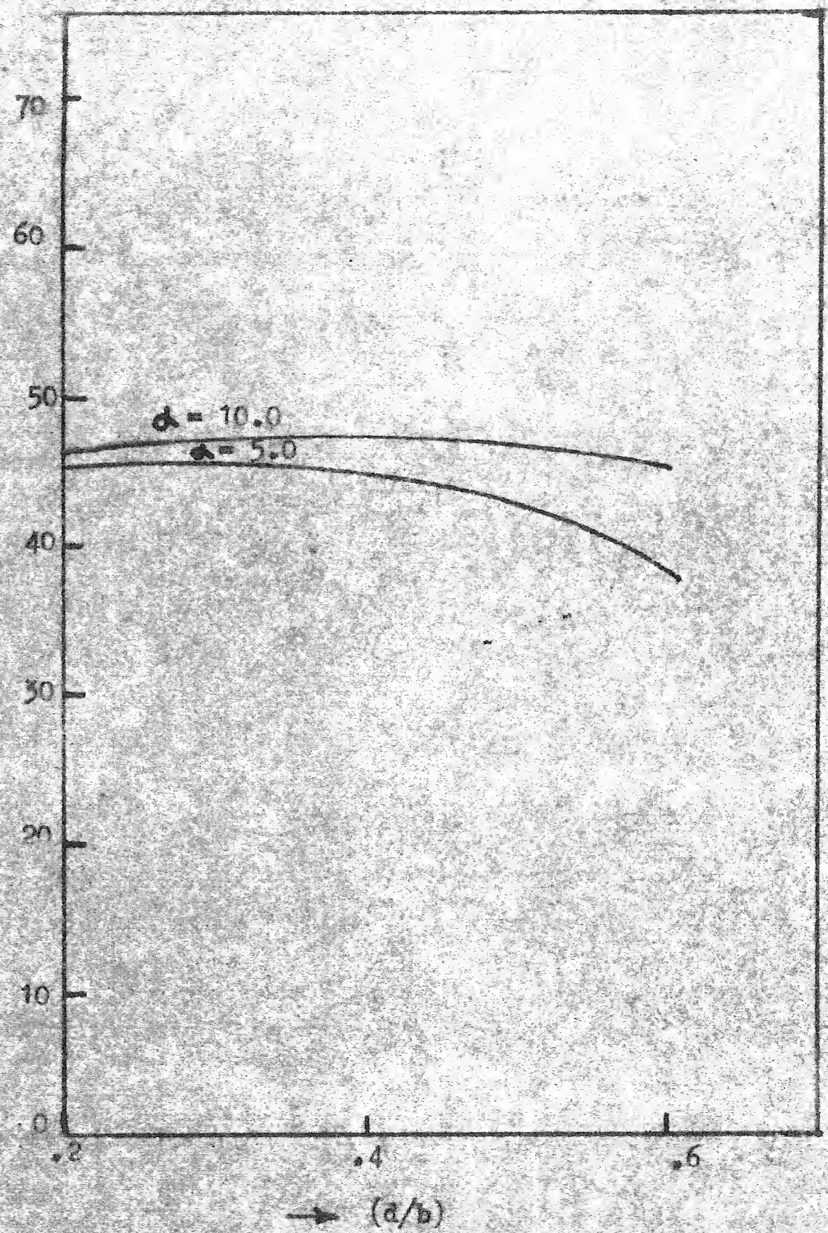


Fig. 4.14 Effect of Position of Joint on Pressure Head at D for, $\mu = 0.5$

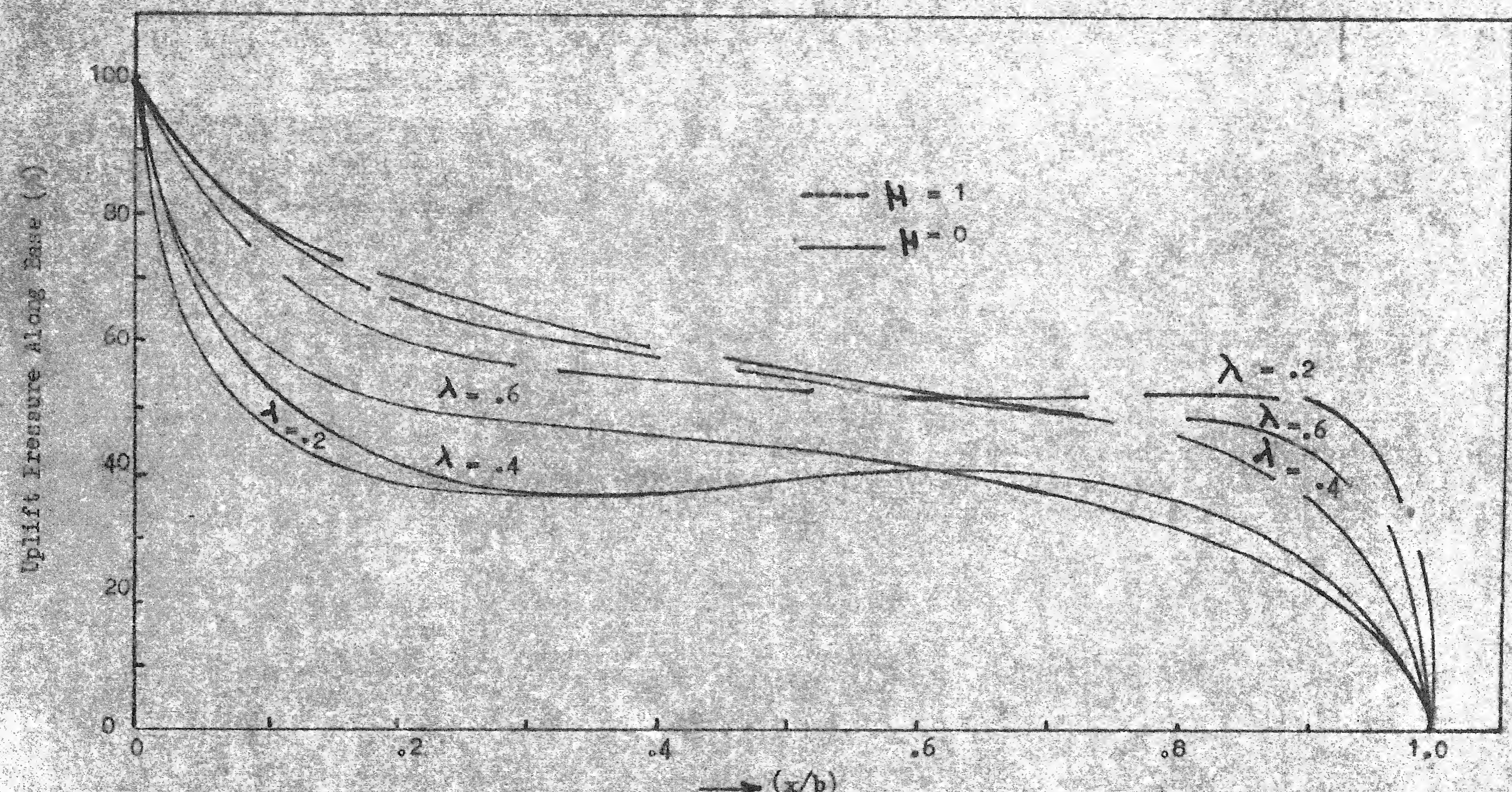


Fig. 4.15 Distinction of Uplift Pressure along the base of weir for $\alpha = 5.0$.

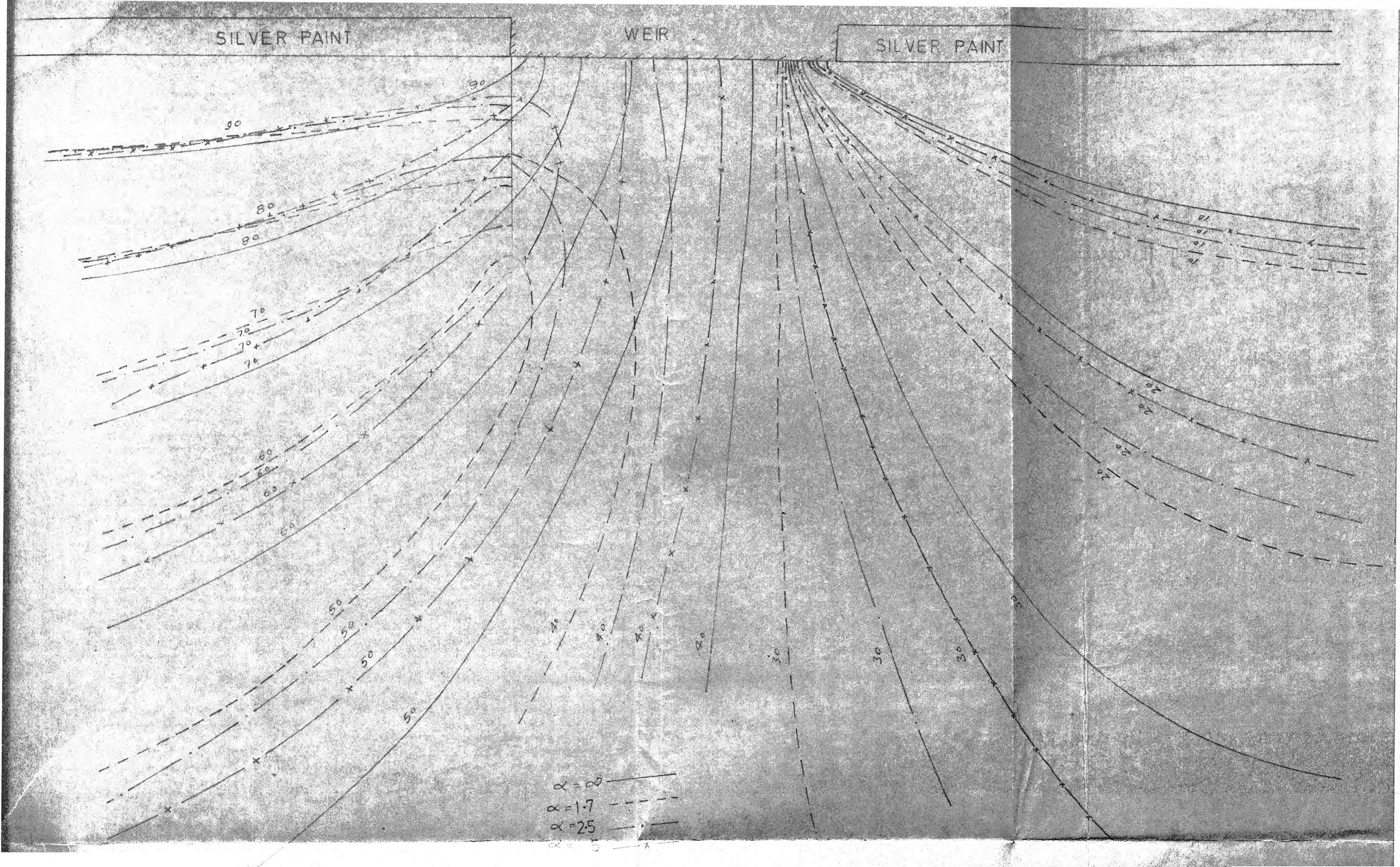


Fig. 4.16 Effect of α on equipotential line for $\eta = 0.8$

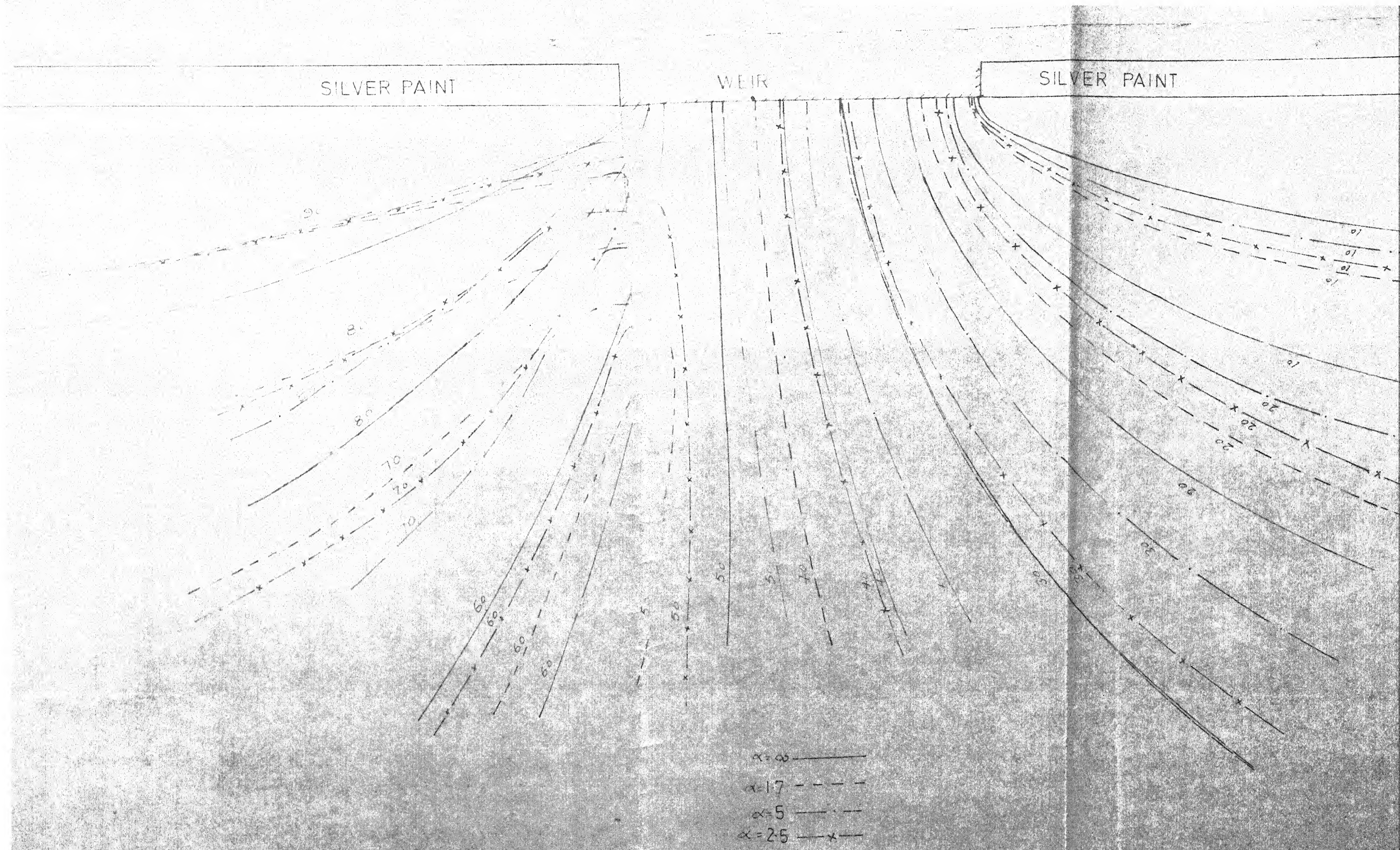


Fig. 4·17 Effect of α on equipotential lines for η = 0·2

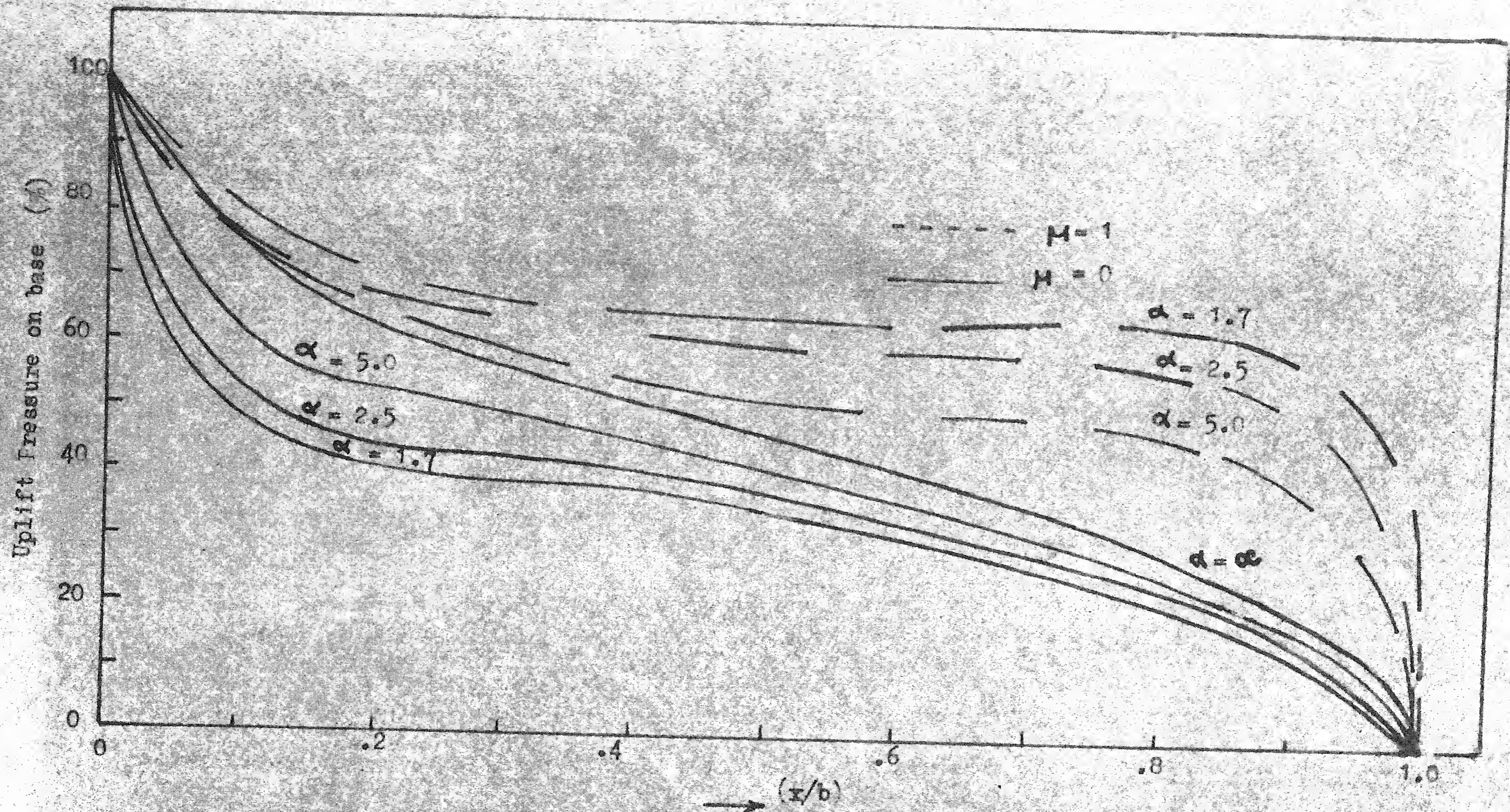


Fig. 4.18 Distribution of Uplift Pressure along the base of the weir for $\eta = 0.2$

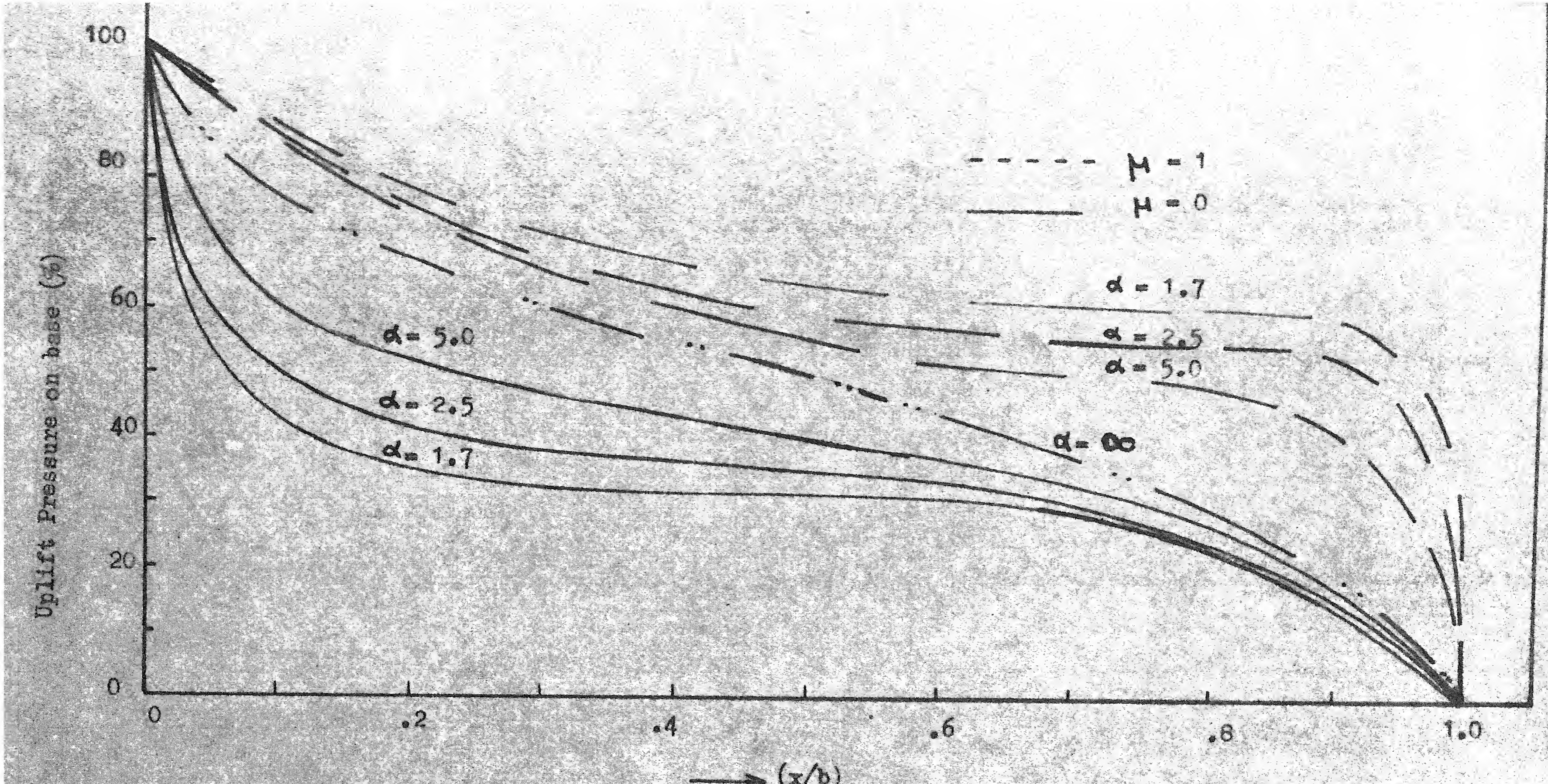


Fig. 4.19 Distribution of Uplift Pressure along the base of the weir for $\eta = 0.4$

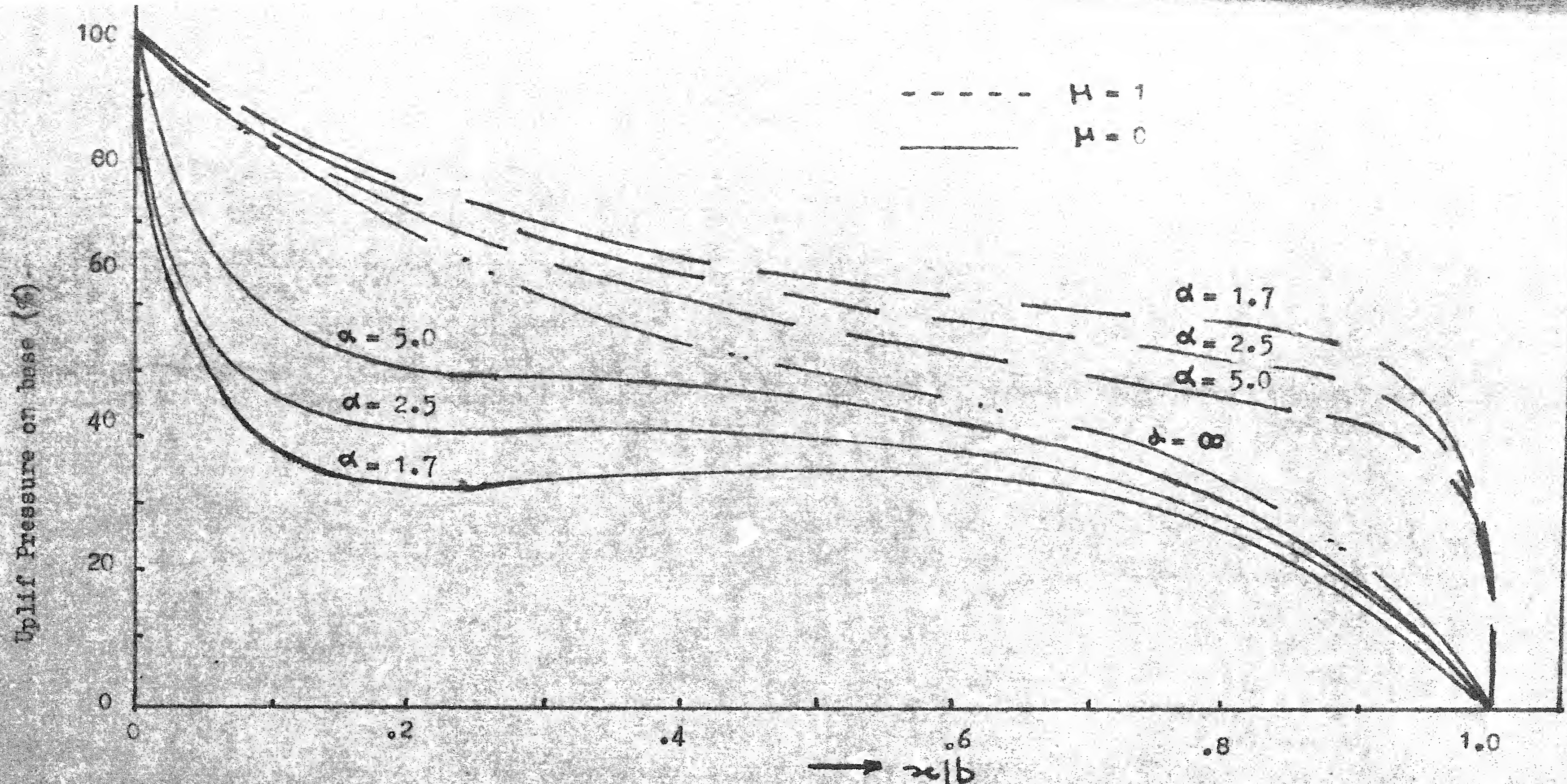


Fig. 4.20 Distribution of Uplift Pressure along the base of the weir for $\eta = 0.8$

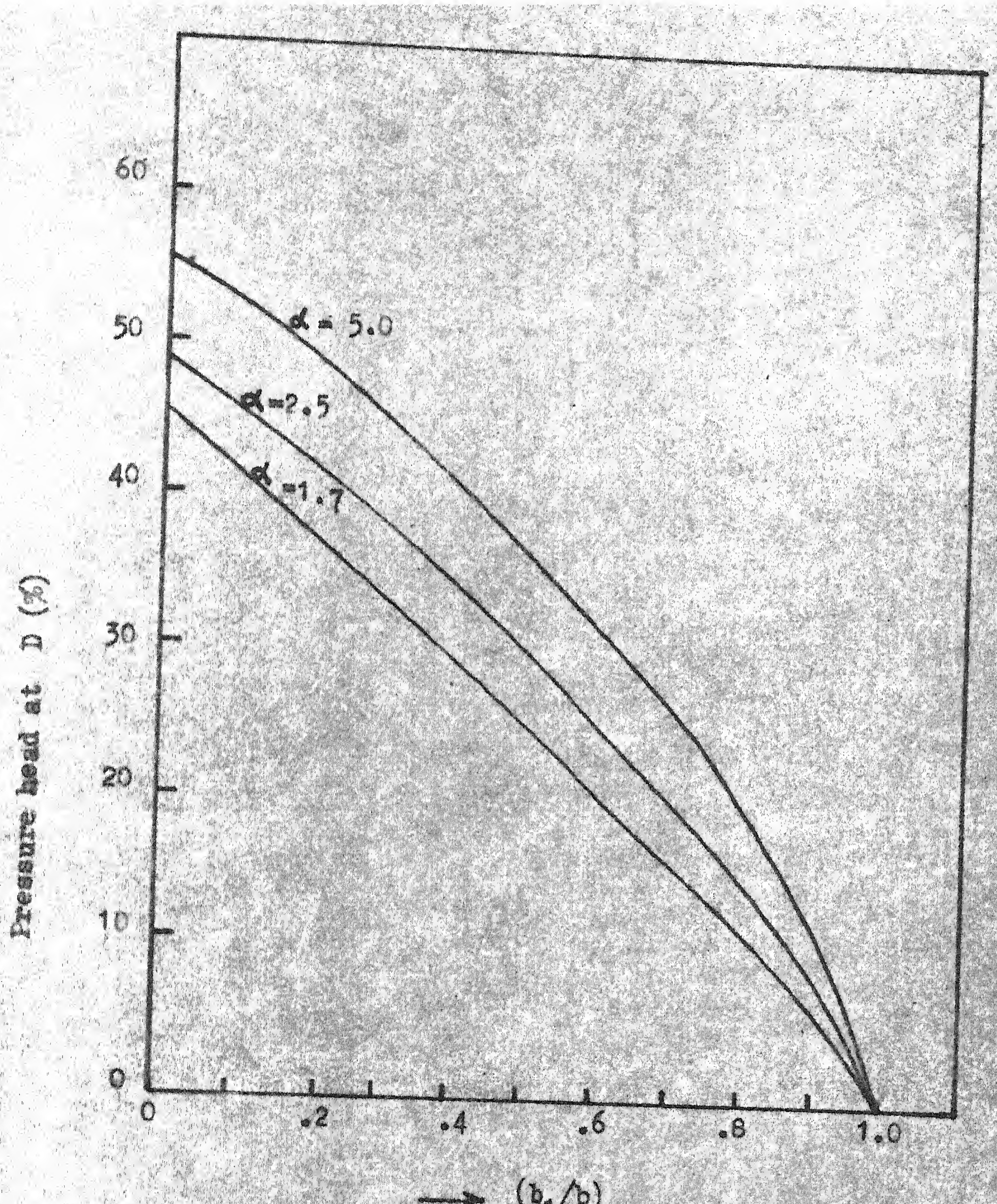


Fig. 4.21 The Effect of Position of Sheet Pile on Pressure Head at D for $\eta = .2$

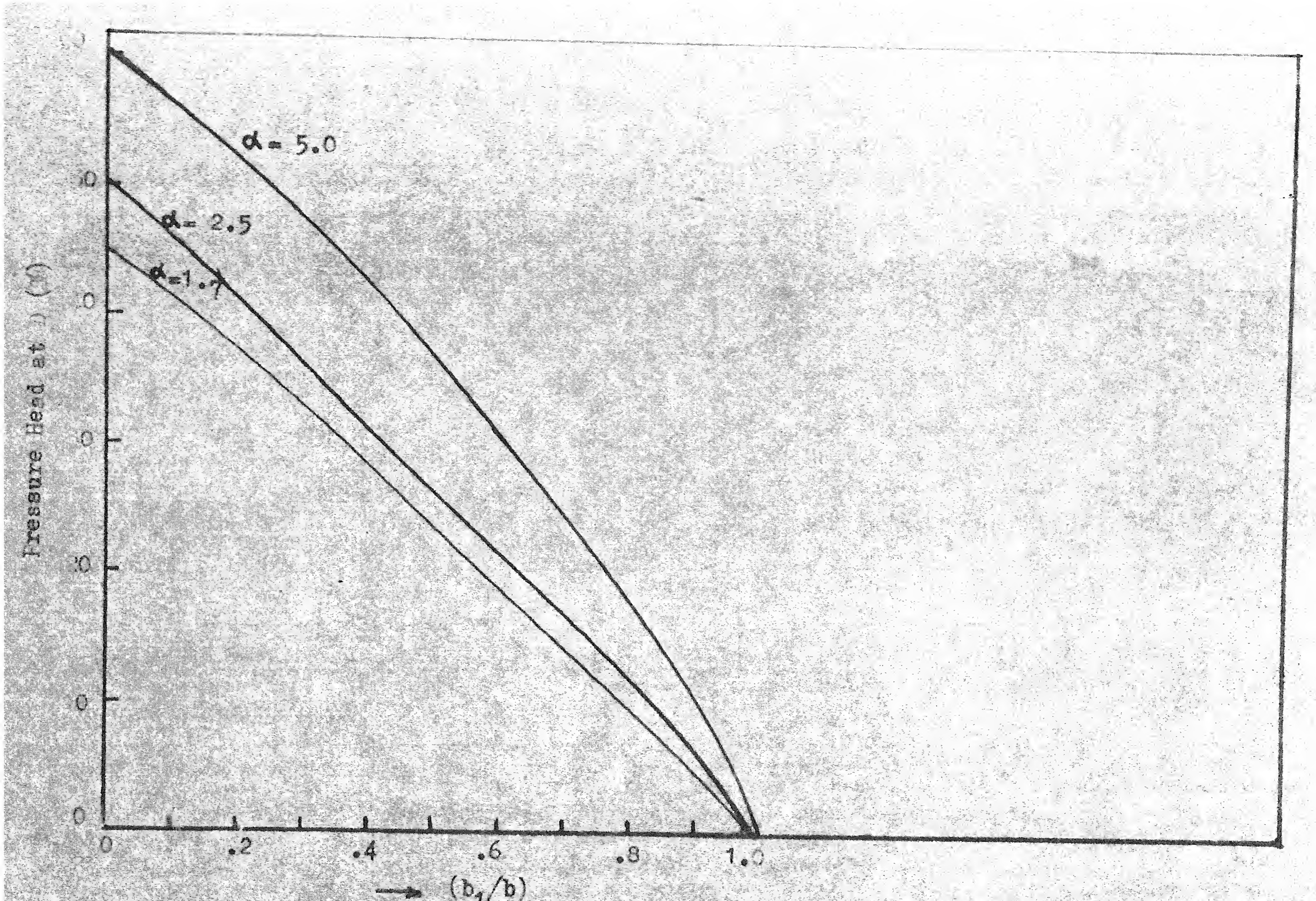


Fig. 4.22 The Effect of Position of Sheet Pile on Pressure Head at D for $\eta = .4$

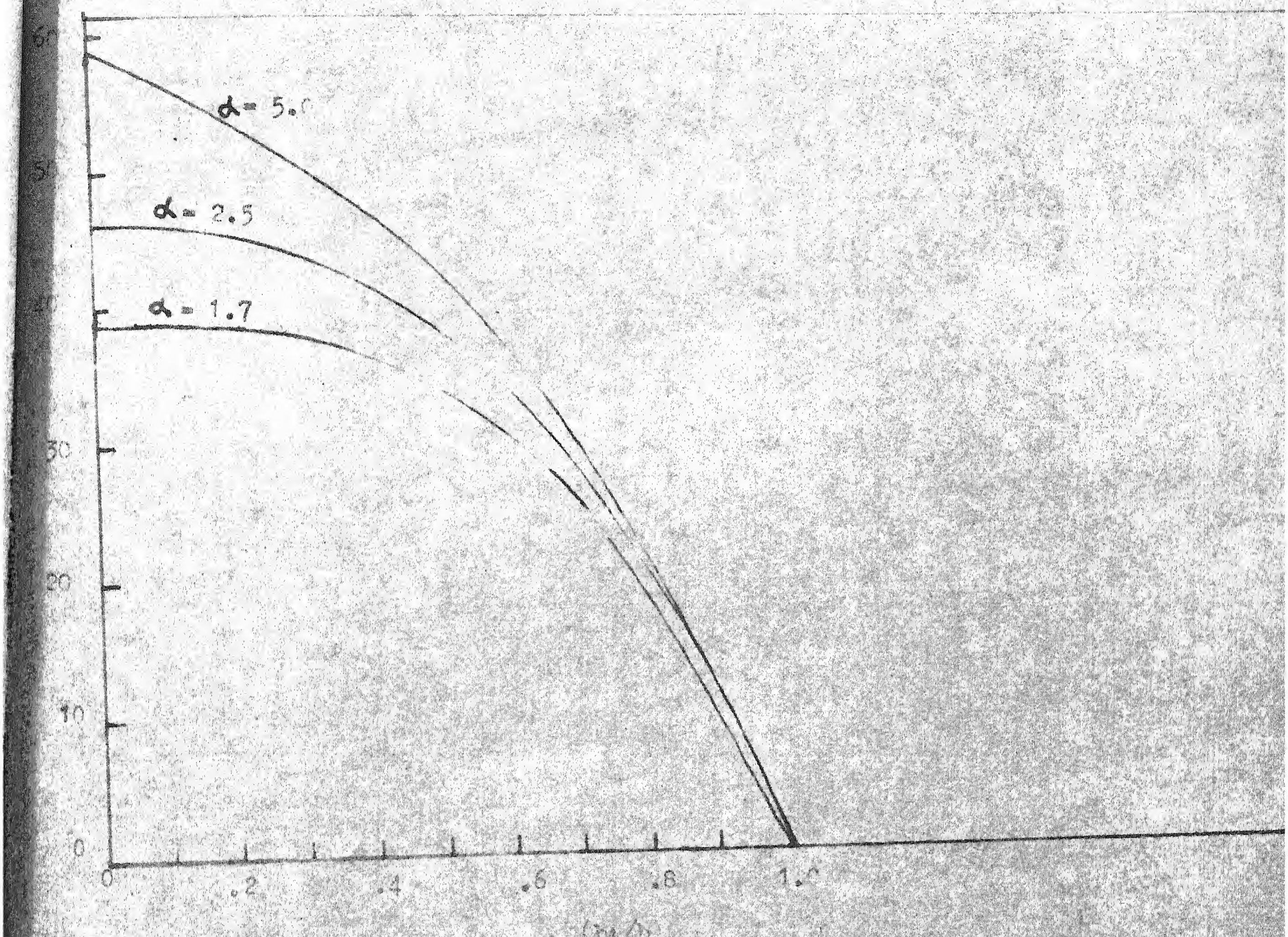


Fig. 3.25 The Effect of Position of Sheet pile on Pressure Head at D; η = 0.8

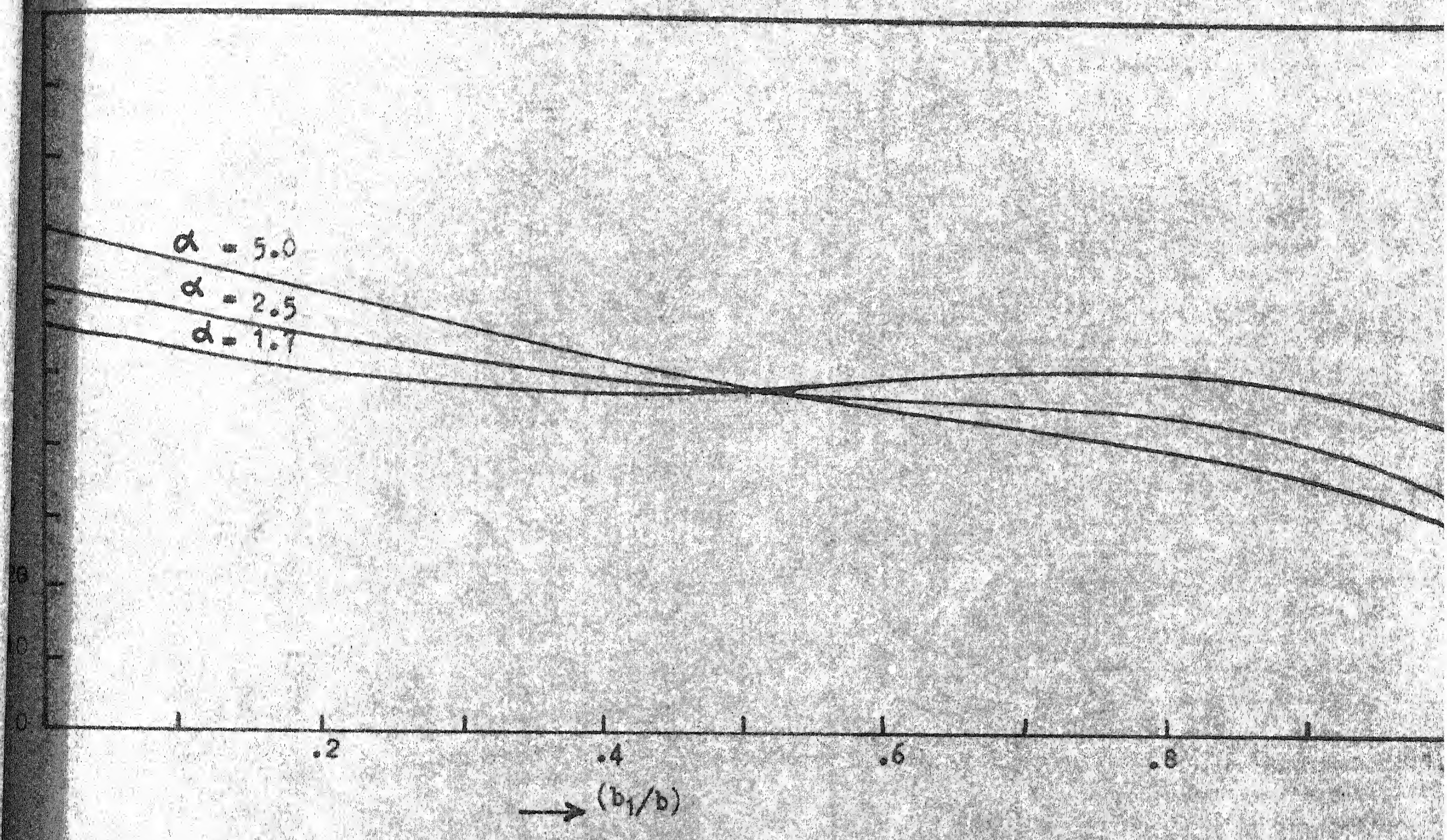


Fig. 4.24 The Effect of Position of Sheet Pile on Pressure Head at C for η = 0.

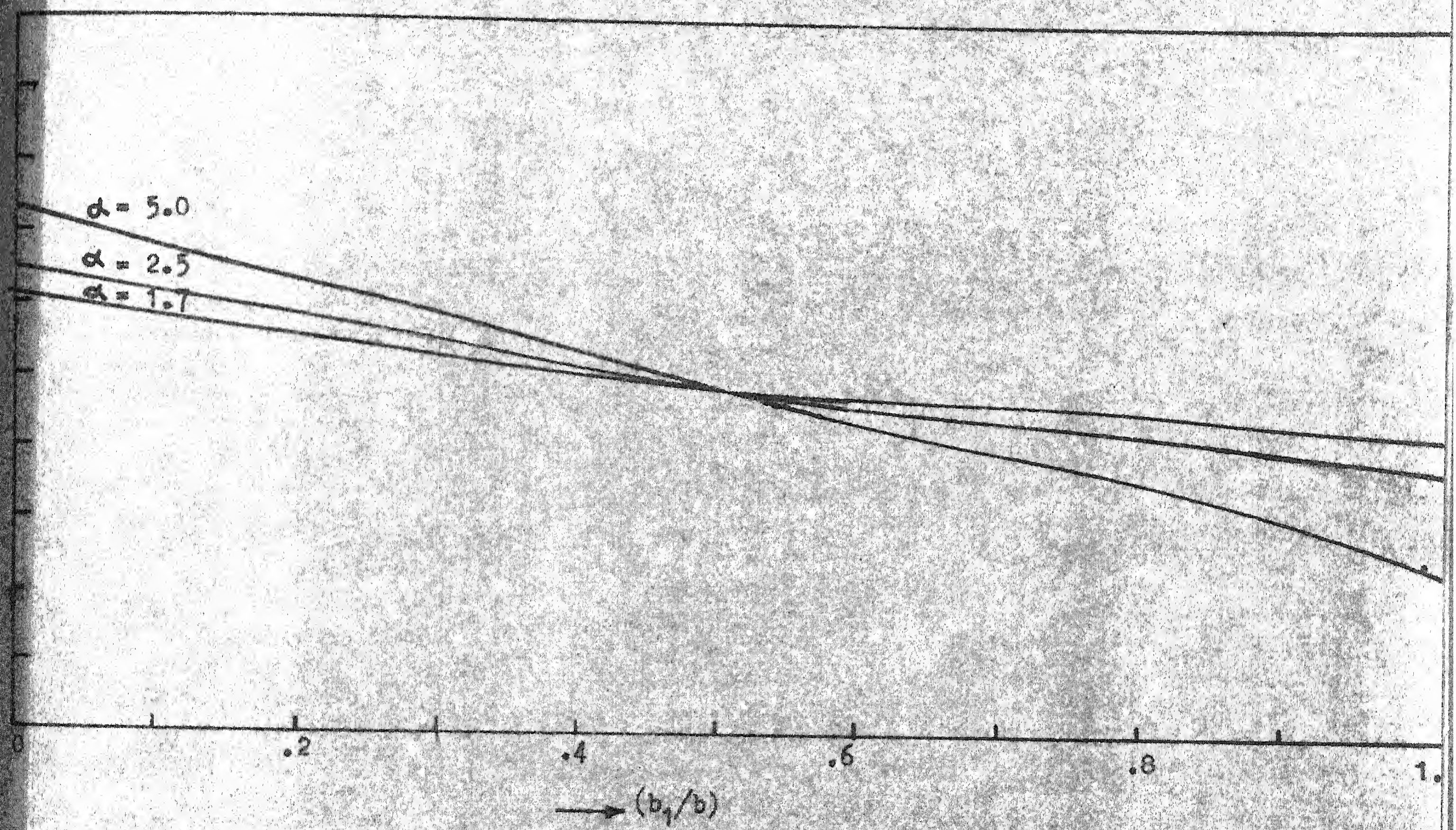


Fig. 4.25 The Effect of Position of Sheet Pile on Pressure Head at C for η = 0.4

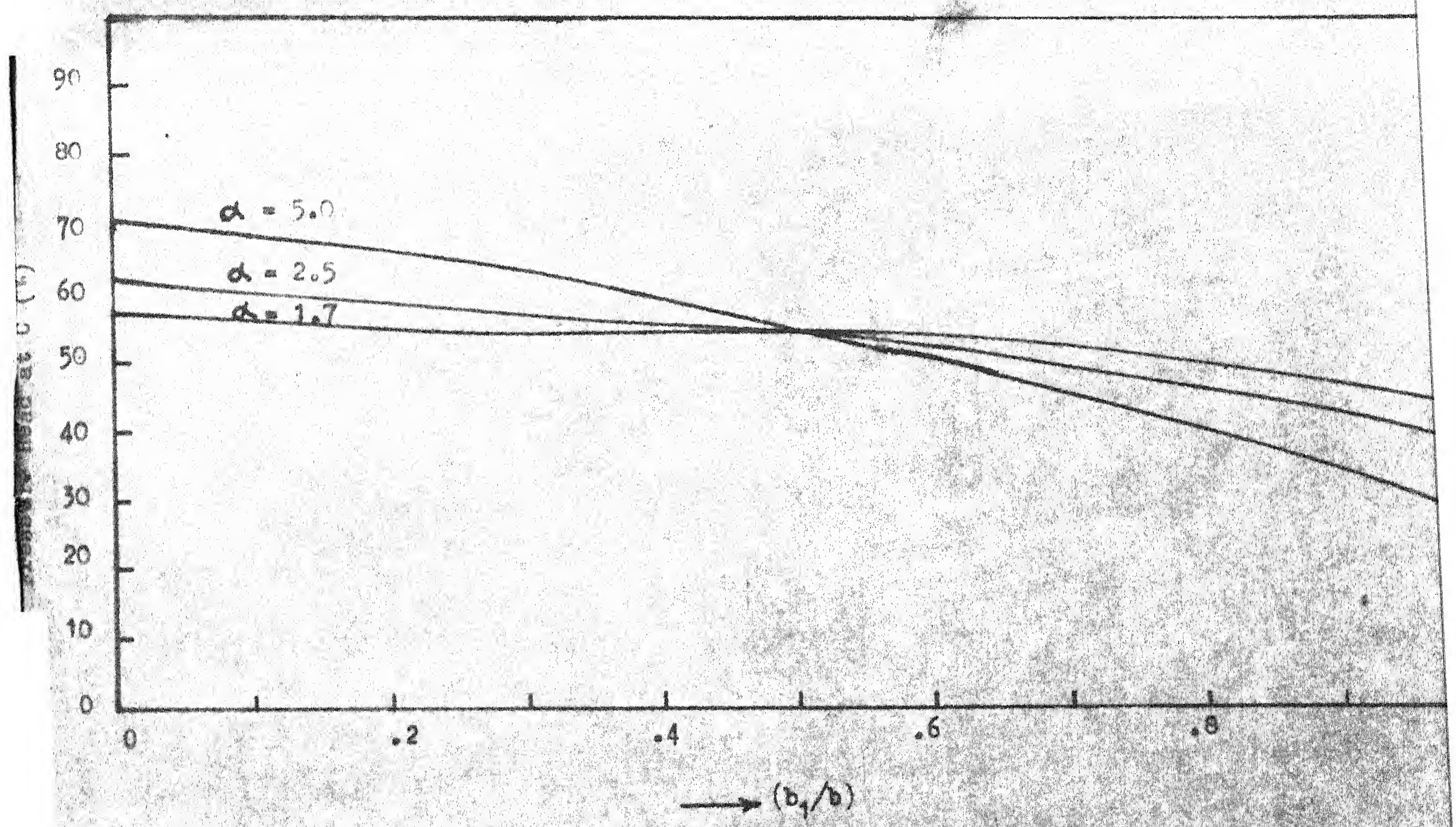


Fig. 4.26 The Effect of Position of Sheet Pile on Pressure Head at C for η = 0.8

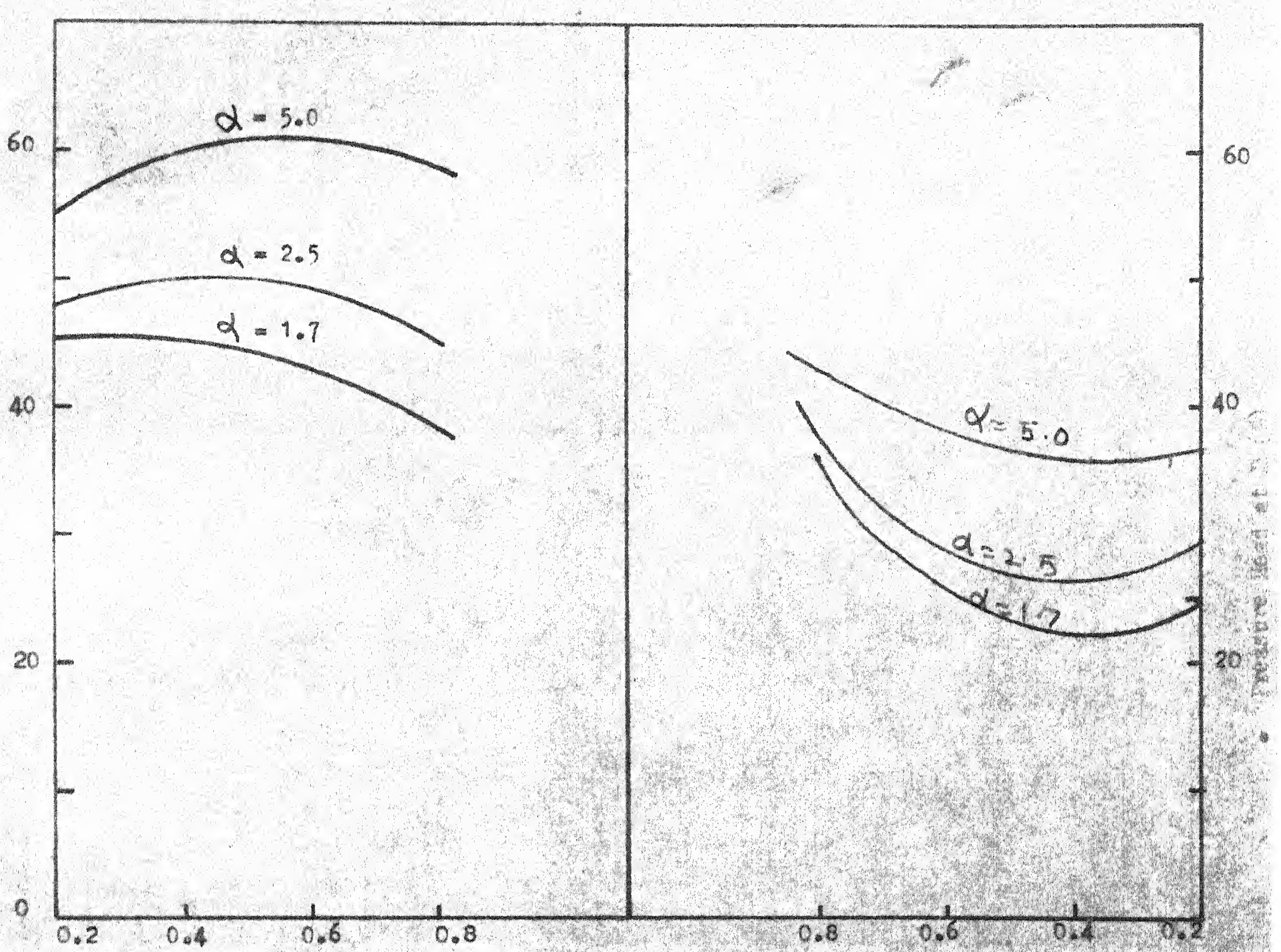


Fig. 4.27 Fig. 4.28

Fig. 4.27 and 4.28 The Effect of Position of Joint on Pressure Head at D for μ = 0.0 and 0.5 respectively.

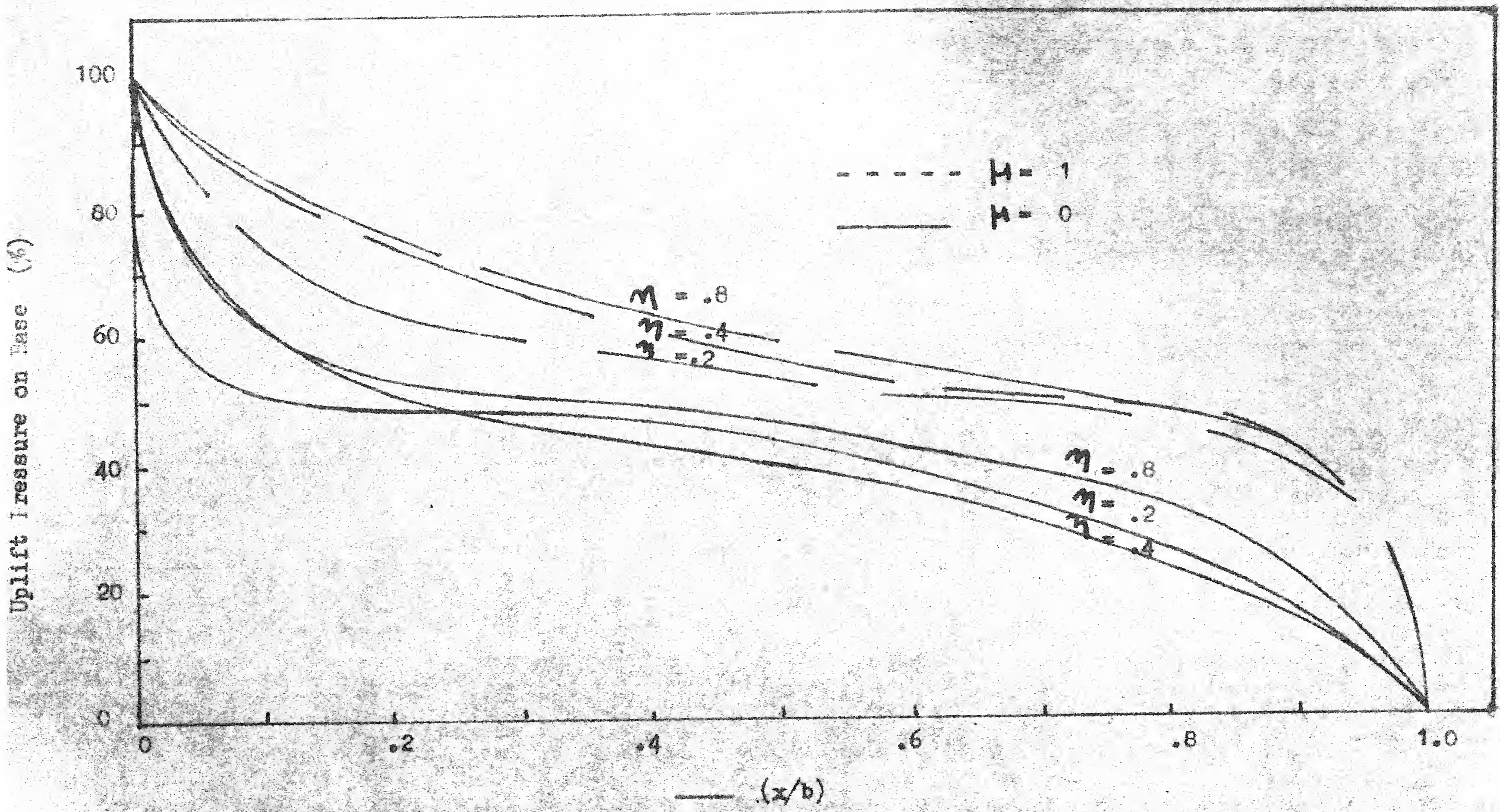


Fig. 4.29 Distribution of Uplift Pressure Along the Base of the Weir for $\alpha = 5.0$.

CHAPTER 5

EFFECT OF LOWER FLOW BOUNDARY AND ANISOTROPY OF THE POROUS MEDIA ON THE UPLIFT PRESSURE FOR A WEIR WITH A SINGLE VERTICAL SHEET PILE:

5.1 GENERAL:

The confined steady flow for a weir with a vertical sheet pile resting on homogeneous mass of infinite extent has been investigated independently by Khosla et al (1954) and Pavlovsky (1933) (vide Harr (1962)). For the same problem treating the porous media to be of finite depth and underlain by a horizontal impervious layer, solutions have been presented by Pavlovsky and Muskat (vide Harr, 1962). Instead of assuming the underlying stratum to be impervious, Grinsky (vide Harr, 1962), presented analysis taking the stratum to be a pervious or a draining one. In all these cases, the flow domain was assumed to be isotropic and homogeneous. However, in many instances pervious stratum through which flow is taking place is anisotropic in nature. Experimental investigations of Graton and Fraser (1935), Arnovici (1947), Reeve and Kirkhan (1951) Mansur and Dietrick

(1965), and Boulton (1970) clearly show the marked evidence of anisotropic nature of permeability. Eventhough, this fact is long since recognized not much work has been done in this area only recently Reddy and Misra (1972) analysed many problems of confined flow taking into account anisotropic nature of soil. However, there are many cases for which no solutions exist and as such in the present investigation results are presented of an electrical analog study dealing with weirs resting on anisotropic soil and for various flow boundary conditions.

5.2 STATEMENT OF THE PROBLEM:

Problems concerned with anisotropic porous media are invariably solved by transforming the actual flow domain into a fictitious isotropic region by a suitable coordinate transformation. The required scale transformation can be derived from the equation of continuity as follows (Harr, 1962).

The equation of continuity for a two dimensional steady flow is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (5.1)$$

where

u, v = discharge velocity in x and y directions, respectively.

From the generalised Darcy's law the relationship between discharge velocity and hydraulic gradient can be written as:

$$u = -(K_x) \left(\frac{\partial h}{\partial x}\right) \qquad (5.2)$$

and $$v = -(K_y) \left(\frac{\partial h}{\partial y}\right) \qquad (5.3)$$

in which:

K_x, K_y = principle coefficient of permeability in x and y directions, respectively.

$h = p/\gamma_w + y$;

p = pressure,

γ_w = unit weight of water and

x, y = co-ordinate axis.

Substitution of values of u and v in equation 5.1 leads to:

$$K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} = 0 \qquad (5.4)$$

Putting $X = x\,(K_y/K_x)^{1/2}$ eqn. 5.4 reduces

$$\frac{\partial^2 h}{\partial X^2} + \frac{\partial^2 h}{\partial y^2} = 0 \qquad (5.5)$$

Alternatively, substituting $Y = y\,(K_x/K_y)^{1/2}$ transforms eqn. 5.4 into:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial Y^2} = 0 \qquad (5.6)$$

Thus, by choosing either of the above transformations a homogeneous an isotropic region can be converted into a fictitious isotropic region for which the laplace's equation is valid.

Based on above general transformations the following example vide Harr (1962) shows how a weir with a vertical sheet pile gets transformed into an inclined sheet pile in fictitious isotropic domain.

Fig. 5.1 represents a cross-section through a row of impervious sheet piles. founded in an anisotropic base. The permeability characteristics of the base soil is given by an ellipse of permeability inclined at 45° to the horizontal with $k_1 = 4k_2$. To get the section transformed, a coordinate system, parallel to the main axis of elipse is established through some point in the flow domain (point 2 Fig. 5.1). The boundaries of the fictitious flow region, wherein Laplce's equation is valid are then obtained by multiplying the perpendicular distances from the boundaries of the natural flow domain to the reference axes by the approximate scaling factor. In Fig. 5.1(a) all the distances parallel to 1-1 axis were reduced by $(k_2/k_1)^{1/2}$. The primed numbers locate the vertices of the transformed section. Once the solution has been obtained for the transformed section (the transformed flow net is given in Fig. 5.1(b), the solution for natural boundaries (Fig.51.~~20~~(c) can be obtained by applying the inverse of scaling factor $(k_1/k_2)^{1/2} = 2$). It is clear from Fig. 5.1(b) that in general, for general anisotropic case the vertical sheet pile becomes an inclined one in the transformed equivalent isotropic region.

In addition to the localised non-homogeneity and anisotropy of porous media another parameter which influences considerably the confined flow behaviour is the nature of flow boundary. Because of its importance many flow boundaries have been considered by various investigators in their analysis and Fig. 5.2. summarises the various cases for which theoretical solutions are available. It is seen from this figure that most of the earlier solutions were based on the assumption of finite porous media underlain either by impervious or draining stratum. only recently Reddy, Mishra and Sitramiah (1972) presented the theoretical solutions wherein the impervious flow boundary was considered as inclined to the horizontal surface.

Electrical analogy method has been also used to study the effect of nature of lower impervious boundary as affecting the uplift pressure. Fig. 5.3 summarises the cases studied by this technique by various investigators.

However, in many field situations the lower flow boundary is neither horizontal nor inclined to the horizontal stratum at one inclination. One typical field cross-section shown in Fig. 5.3, which is quite of common occurrence, indicates that it is necessary to investigate the case of a weir resting on pervious

stratum underlain by an impervious stratum with sloping impervious strata downstream (Fig. 5.4). Also this case is most general in the sense that if the distance m (Fig. 5.4) is very large as compared to other dimensions, the case corresponds to that of a finite porous media underlain by impervious strata and when d is very large as compared to other dimensions, the case corresponds to that of a inclined impervious boundary. Simulating such a flow boundary results are obtained for a weir with one sheet pile resting on anisotropic soil. As has been explained.earlier the vertical sheet pile becomes an inclined one in the transformed section and as such in electrical analogy study the sheet pile is taken to be an inclined one to account for anisotropy of the soil. It should be noted that the results thus obtained for this case and subsequent cases are for the transformed isotropic zone.

The second flow boundary which has been considered is shown in Fig. 5.5, where it is assumed that a sloping draining stratum exists at downstream of the weir. Once again the medium through which flow is taking place is assumed to be anisotropic. It should

be noted here that the present case falls at the other extreme for which Misra et al presented the solution. It is obvious that the solution of Misra and the results of the present study serve ~~same~~ as two bounds to a two layered soil system seperated by the inclined flow boundary.

The last flow boundary condition considered in the analysis is schematically shown in Fig. 5.6. Because the underlying and the sloping stratum are assumed to be free draining the analysis of this case in conjunction with results of 1st case would serve as two bounds for a layered soil system separated by the flow boundary.

The different parameter that are varied in all the above cases are defined in Figs. 5.4, 5.5 and 5.6. For presenting the results in non-dimensional form following dimensionless parameters are defined for various cases studied.

$\beta = m/b$

$\alpha = b/s$

$\lambda' = d'/b$

$\mu = b_1/b$

$\alpha' = b/s'$

where m = distance of the inclined pervious or impervious boundary from toe of the weir.

b = base width of weir,

s' = embedded length of inclined sheet pile,

s = embedded length of vertical sheet pile ,

d' = distance of pervious or impervious stratum from base of the weir.

γ = angle in degree made by embedded length of sheet pile with horizontal boundary as shown in Figs. 5.4 and 5.5,

b_1 = distance of sheet pile from heal of the weir.

The value of γ will obviously depend upon the inclination of the axis of major or minor principal permeability with horizontal axis. Tables 5.1, 5.2, and 5.3 summerise. the values of various variables as defined above for which results have been obtained in the present study.

5.3 RESULTS AND DISCUSSION OF RESULTS:

Case (i) weir resting on a finite anisotropic pervious soil underlain by an impervious stratum with inclined sloping impervious boundary downstream.

To study the effect of anisotropy of the soil, the angle γ was varied from 30° to 120° with 30° interval. For studying the effect of lower flow boundary λ' and β were taken equal to 1.0. These values have been considered in the analysis in preference to many other combinations as the primary aim was to study the presence of such a flow boundary in the vicinity of location of weir. However the variation in geometry of the lower flow boundary is achieved by taking $\theta = 60^{\circ}$ and 30°.

Figs. 5.7 through 5.14 show the variation of uplift pressure along the base of the weir for various inclinations of sheet pile and downstream sloping impervious boundary. For studying the effect of position of sheet pile as affecting the uplift pressure distribution two extreme cases namely (i) pile

at upstream end and (ii) pile at downstream end have been studied. Fig. 5.7 shows a typical variation of uplift pressure at base for $\theta = 60^0$, $\gamma = 30^0$ $\lambda = 1.0$ and $\beta = 1.0$ its can be expected it is seen that with increase in length of the sheet pile there is considerable decrease in uplift pressure for a sheet pile located at upstream and whereas the trend is exactly reverse in case of pile located at the downstream end. Similar trends are observed in all the Figs. 5.8 through 5.14. The effect of anisotropy of soil as affecting the uplift pressure at the base is shown in Fig. 5.15(a). It is seen from this figure that the uplift pressure distribution is considerably influenced by the inclination of major axis of the ellipse of anisotropy with horizontal direction. The effect of λ, on uplift pressure is shown in Figs. 5.15(b) and 5.15(c) for $\alpha = \infty$, $\beta = 1.0$ and $\theta = 60°$ and $30°$ respectively. For $\theta = 60°$ the results exhibit similar trend as in the absence of sloping impervious stratum (Harr 1962). However, when $\theta = 30°$, implying thereby inclined impervious boundary more closer to weir, the trend is changed in relation to that for $\theta = 60°$.

Case (ii) Weir resting on infinite anisotropic porous medium with sloping pervious boundary.

To study the effect of inclination and the location of pervious sloping boundary results have been obtained for $\theta = 15^{\circ}$, 30° and 60° and $\beta = 1.65$, 1.0 and 0.5 for the case of a weir with no sheet pile (i.e. $\alpha = \infty$). These results are presented in Figs. 5.16 through 5.18. Fig. 5.16 shows the variation of the uplift pressure distribution along the base of the weir for $\beta = 1.65$ for various values of θ. As can be seen from the graph, the pressures decrease considerably as θ decreases. This is obvious, because with nearer pervious boundary, the higher equpotential lines turn more towards the upstream side of the weir thereby decreasing the pressures along the base. Fig. 5.19 shows the effect of θ and β on the pressure head at the center of the weir. It is seen from this figure that with decreasing β and θ there is considerable reduction in the uplift pressure.

Fig. 5.20 through 5.27 show the variation of uplift pressure distribution along the base of the weir for $\gamma = 30^{\circ}$, 60°, 90°, 120° and $\theta = 30^{\circ}$ and 60°. These results are for $\beta = 1.0$ and for $\alpha' = 5.0$, 2.5 and 1.25 and $\mu = 0$ and 1.0. Fig. 5.20 shows a typical variation

of uplift pressure distribution along the base of the weir for $\theta = 30^0$, γ 30^0, $\beta = 1.0$, $M = 0$ and 1.0 and $\alpha' = 1.25$, 2.5, and 5.0. The figure once again brigs out the effectiveness of placing the sheet pile at the upstream end for reducing the uplift pressures. Fig. 5.28 and Fig.5.29 show the effect of location of sheet pile on pressure head at tip of the pile for m/b =1.0 and 0.5 respectively for both $\theta = 60^0$ and 30^0 and for γ equal to 90^0. These results show that with decreasing value of m/b there is considerable decrease in pressure head at the tip of the pile. Comparison of curves for same m/b ratio and α' but with $\theta = 30^0$ and 60^0 shows that with decreasing value of angle θ there is considerable decrease in pressure heads. Similar results for point D (Refer inset diagram) are presented in Fig.5.30and 5.31 exhibiting similar trends.

Case (iii) weir resting on finite pervious anisotropic soil underlain by draining stratum and with inclined pervious boundary downstream:

In the first instance, to study the effect proximity of such a pervious boundary as affecting the uplift pressure, results were obtained for $\lambda = 1.0$ and

0.5 for $\beta = 1.0$, $\alpha = \infty$ and $\theta = 60^\circ$ and 30°. Figs. 5.32 and 5.33 show the corresponding results. It is seen from these figures that with decreasing values of λ' (thereby implying nearer draining stratum) and θ the pressures decrease considerably along the base of the weir. To study the combined effect of anisotropy and lower flow boundary results have been obtained for $\gamma = 30^\circ$, 60°, 90° and 120°, $\beta = 1.0$ and $\lambda = 1.0$. The variation in lower flow boundary being achieved by varying angle θ as 30° and 60°. These results are presented in Figs. 5.34 through 5.41. Fig. 5.34 shows a typical variation of uplift pressure along the base of the weir for $\theta = 60^\circ$ and $\gamma = 30^\circ$ for various values of α and for pile located either at upstream or downstream end. By comparing Fig. 5.34 of this case to Fig. 5.7 of case (i) it is seen that because of the extreme change in the nature of the flow boundary from impervious to a pervious one there is considerable decrease in uplift pressure distribution. Similar trends can be also observed by comparing Figs. 5.8 through 5.14 with 5.35 through 5.41.

TABLE 5.1 IMPERVIOUS STRATUM WITH SLOPING IMPERVIOUS STRATUM.

λ' = 0, 1.0

β (=m/b)	λ' (d'/b)	θ		α or α'
			30°	= 5,2.5,1.25
			60°	= 5,2.5,1.25
			90°	= 5,2.5,1.7
		60°	120°	= 5,2.5, 1.25
1.0	1.0			
			30°	= 5,2.5,1.25
			60°	= 5,2.5,1.25
		30°	90°	= 5,2.5,1.70
			120°	= 5,2.5,1.25

TABLE 5.2 PERVIOUS SLOPING BOUNDARY

5.2(a) No Sheet Pile

θ	β(=m/b)
	.5
60°	1.0
	1.65
	.5
30°	1.0
	1.65
	.5
15°	1.0
	1.6

TABLE 5.2 (b) VERTICAL SHEET PILE

$\mu (= b_1/b) = 0, 0.5, 1.0$

θ	β	α
60°	1.0	5, 2.5, 1.7
	0.5	5, 2.5, 1.7
30°	1.0	5, 2.5, 1.7
	0.5	5, 2.5, 1.7

TABLE 5.2(c) INCLINED SHEET PILE

$\mu = 0, 1.0$ $\beta = 1.0$

θ	γ	α'
60°	30°	5, 2.5, 1.25
	60°	5, 2.5, 1.25
	120°	5, 2.5, 1.25
30°	30°	5, 2.5, 1.25
	60°	5, 2.5, 1.25
	120°	5, 2.5, 1.25

TABLE 5.3 PERVIOUS STRATUM WITH INCLINED PERVIOUS BOUNDARY

$\mu = 0,\ 1.0$

$\beta = 1.0$

$\lambda = 1.0$

θ	γ	α or α'
60°	30°	5, 2.5, 1.25
	60°	5, 2.5, 1.25
	90°	10, 5, 2.5
	120°	5, 2.5, 1.25
30°	30°	5, 2.5, 1.25
	60°	5, 2.5, 1.25
	90°.	10, 5, 2.5
	120°	5, 2.5, 1.25

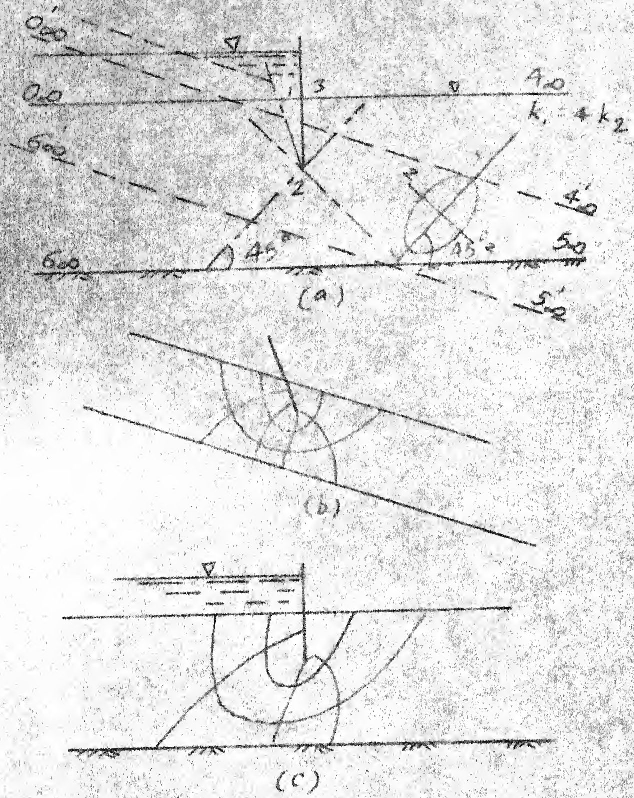
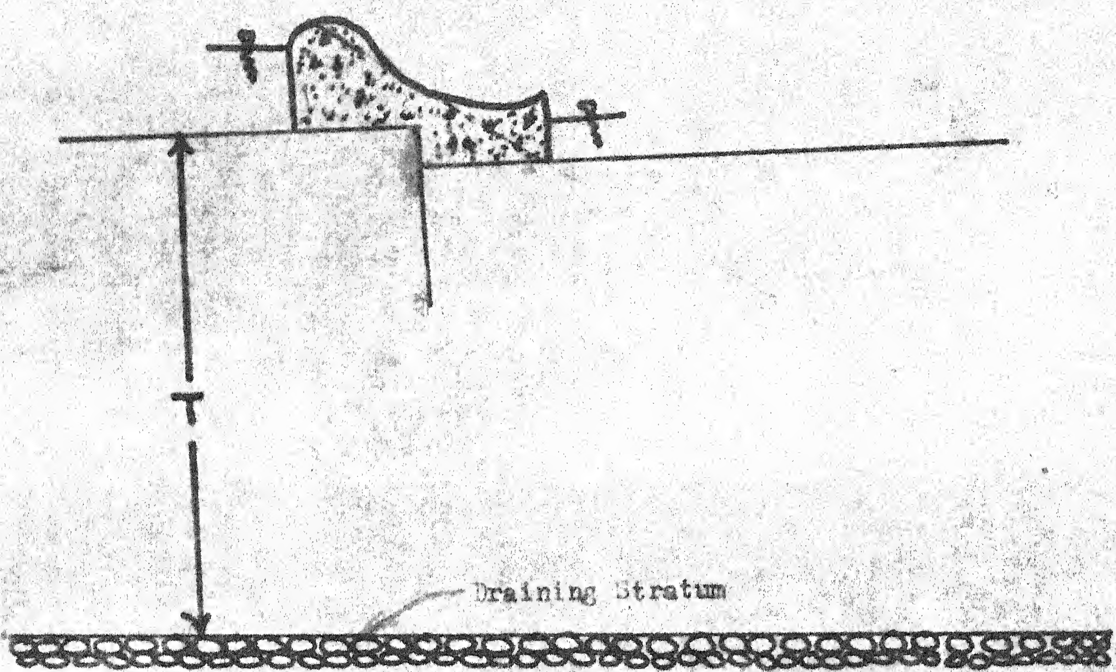


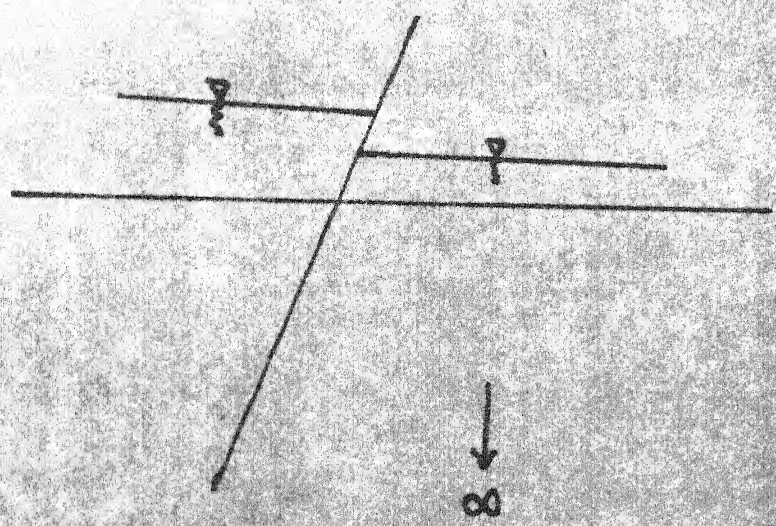
FIG. 51

TRANSFORMATION FOR A VERTICAL SHEET PILE FROM ANISOTROPIC REGION TO AN EQUIVALENT ISOTROPIC REGION



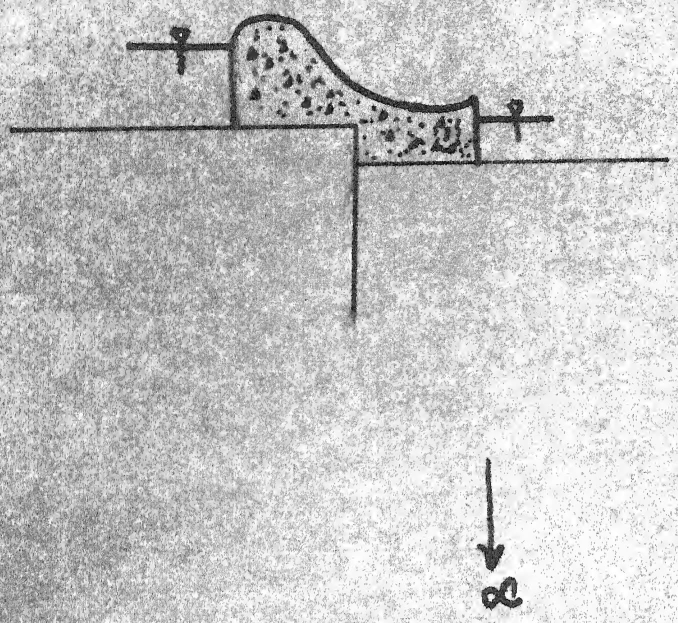
(After Grinisky)

(c) WEIR RESTING ON FINITE POROUS MEDIUM UNDERLAIN BY PERVIOUS STRATUM



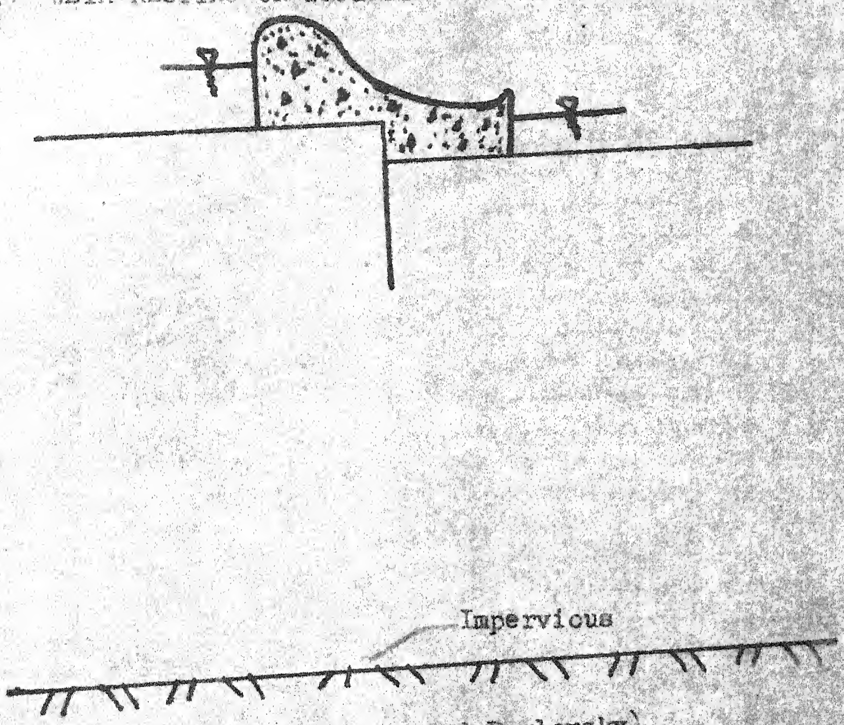
(After Virgin)

(d) Vertical Sheet Pile in Infinite Anisotropic Porous Medium.



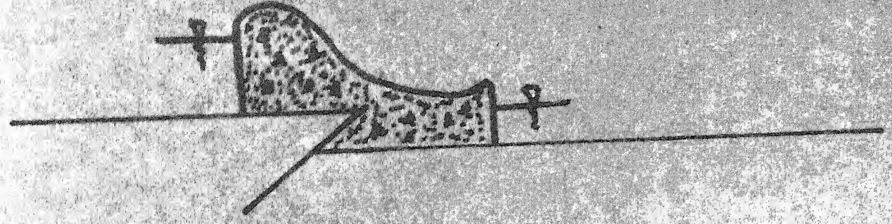
(After Khosla et.al.)

(a) WEIR RESTING ON INFINITE ISOTROPIC POROUS MEDIUM



(After Muskat and Parlovsky)

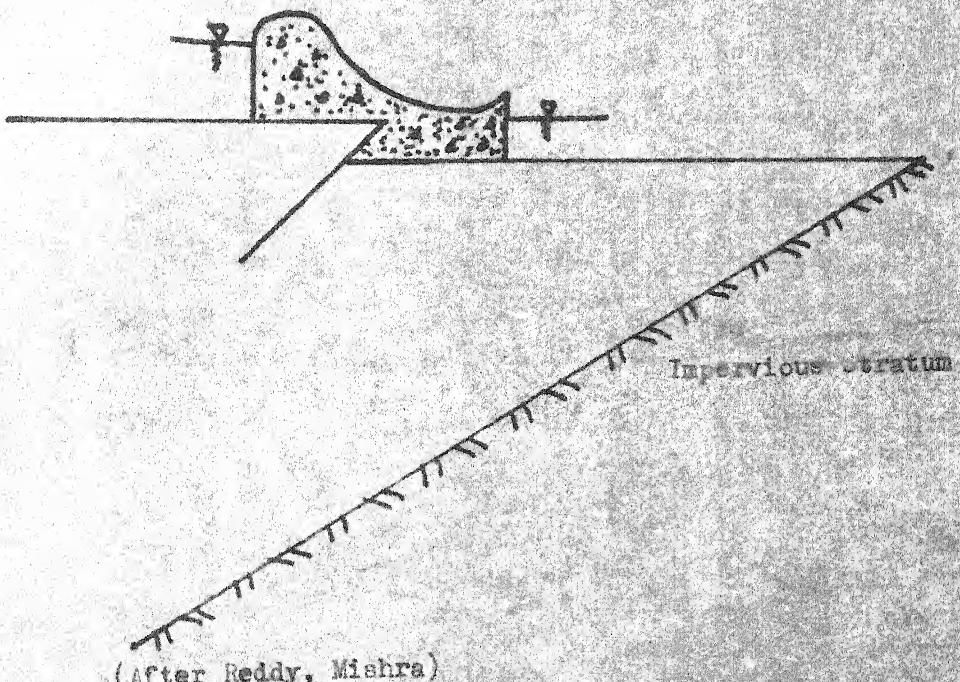
(b) WEIR RESTING ON FINITE POROUS MEDIUM UNDERLAIN BY IMPERVIOUS STRATUM



either impervious or pervious.

(After Reddy, Mishra)

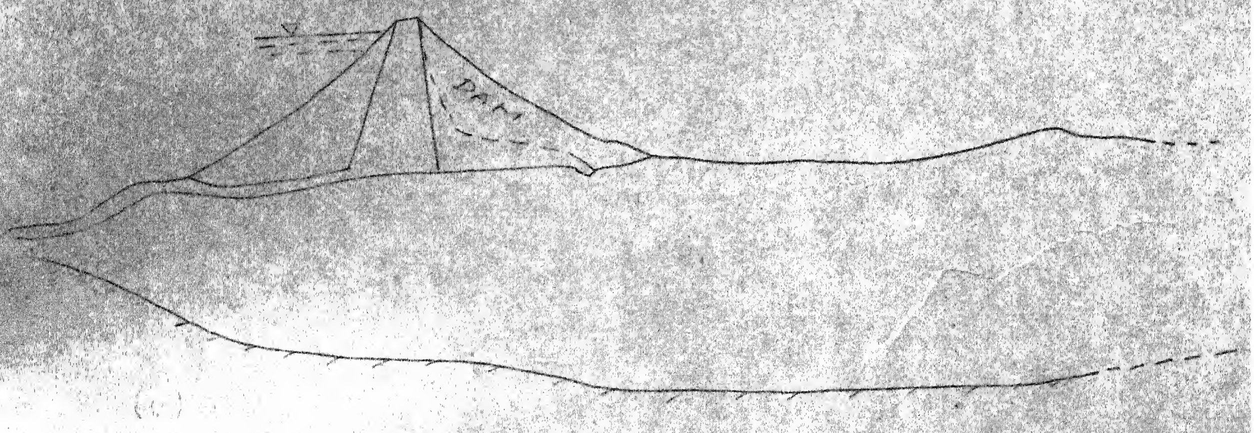
(e) WEIR RESTING ON FINITE ANISOTROPIC MEDIUM UNDERLAIN BY EITHER IMPERVIOUS OR DRAINING STRATUM



(After Reddy, Mishra)

(f) WEIR RESTING ON ANISOTROPIC MEDIUM WITH SLOPING IMPERVIOUS BOUNDARY

FIG. 5.2 SCHEMATIC DIAGRAMS SHOWING VARIOUS FLOW BOUNDARIES FOR WHICH THEORETICAL SOLUTIONS EXIST.



(After Scott)

Fig. 5.3 LOWER FLOW BOUNDARIES CONSIDERED IN ANALOG SOLUTIONS.

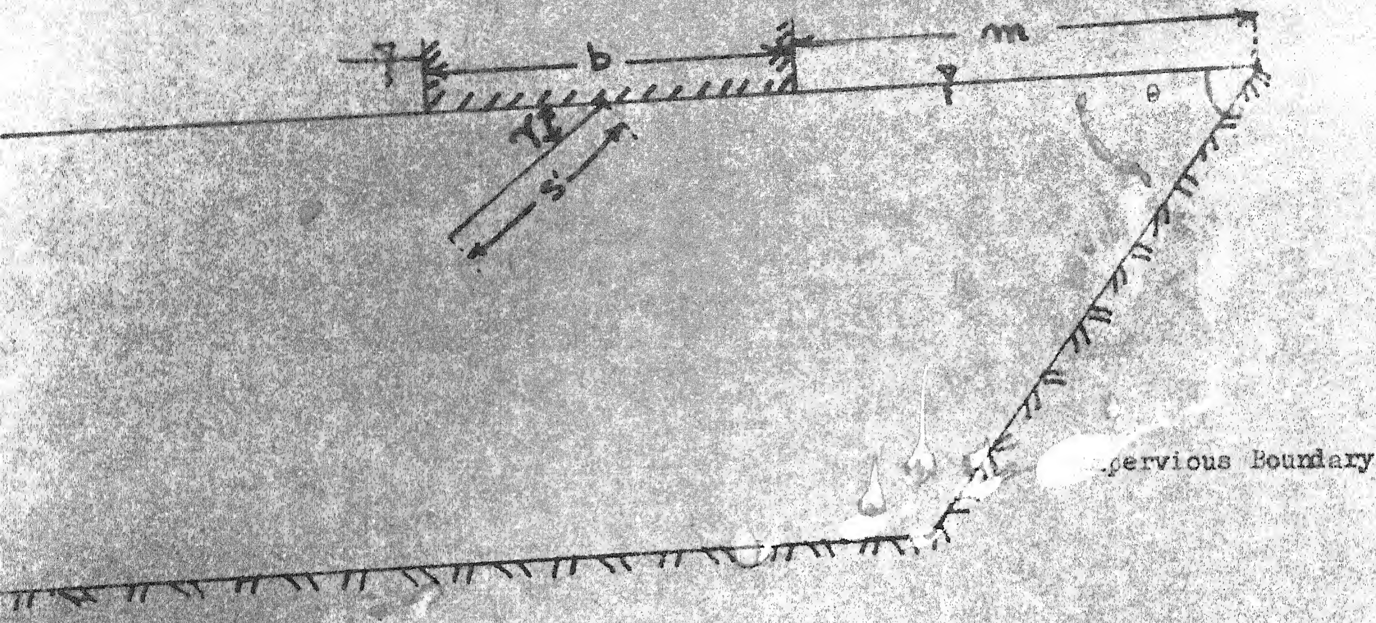


Fig. 5.4 WEIR RESTING ON ANISOTROPIC SOIL WITH INCLINED AND HORIZONTAL PERVIOUS BOUNDARY

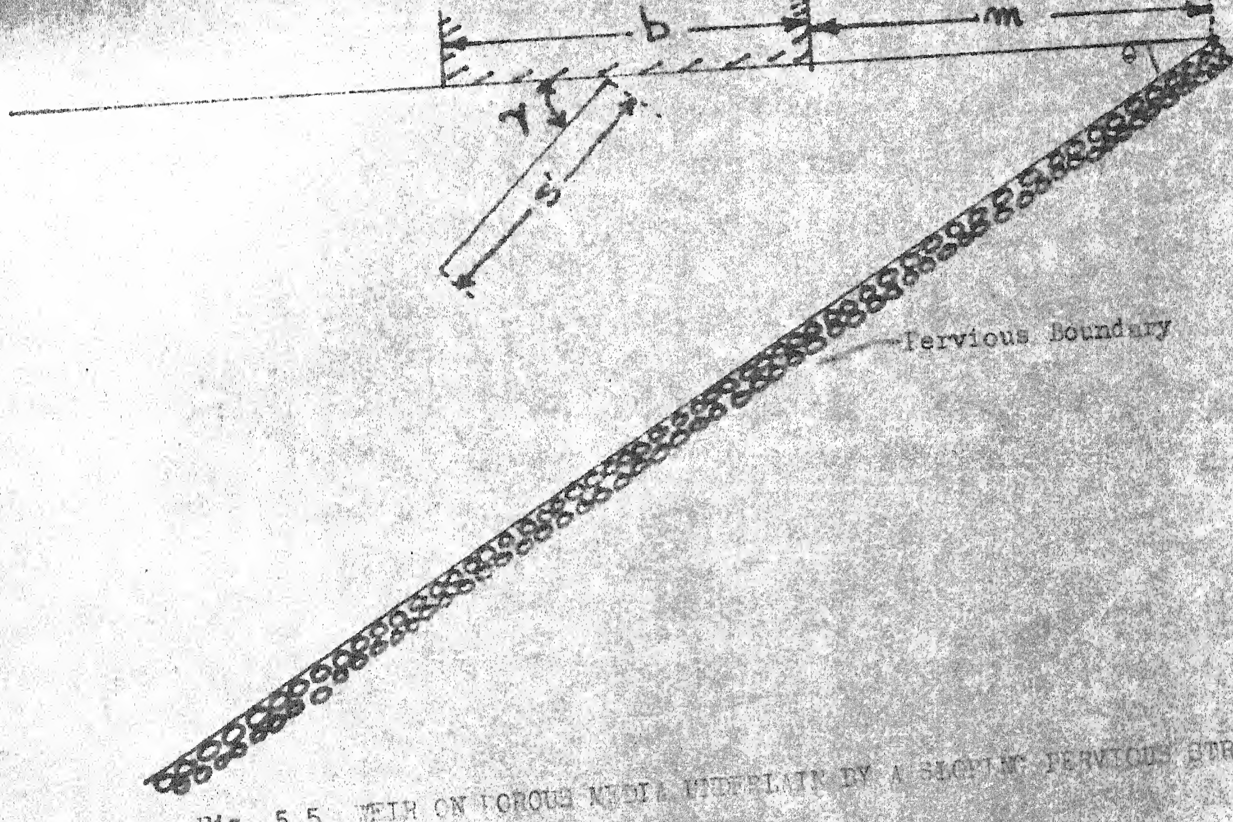


Fig. 5.5 WEIR ON POROUS MEDIA UNDERLAIN BY A SLOPING PERVIOUS STRATUM

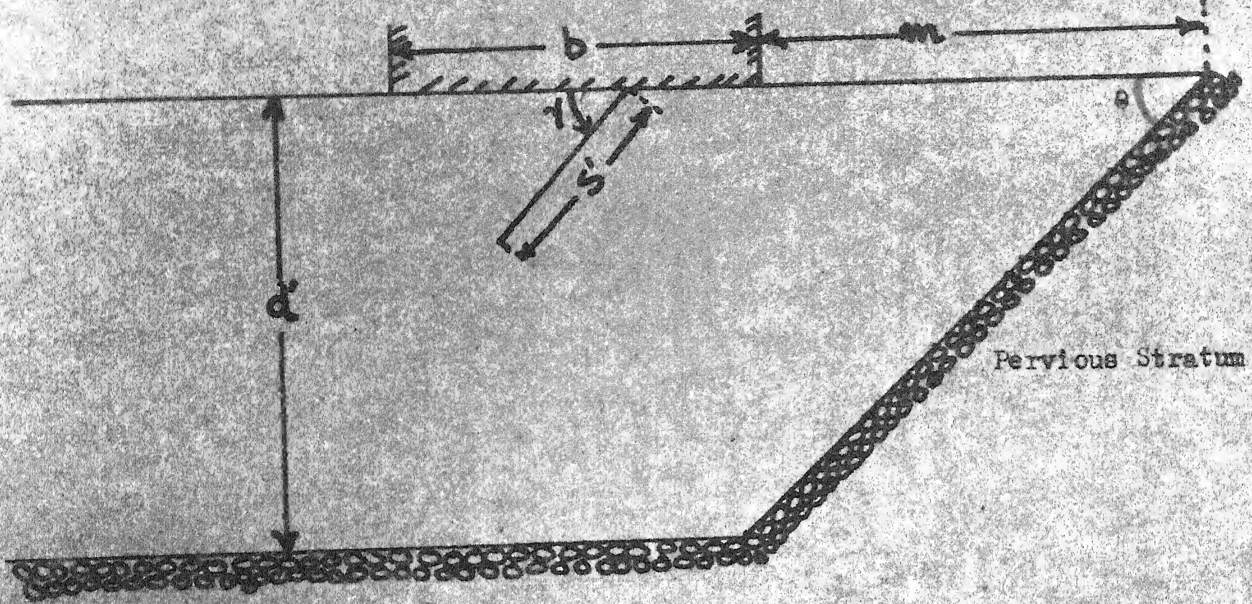


Fig. 5.6 WEIR ON POROUS MEDIA UNDERLAIN BY INCLINED AND HORIZONTAL PERVIOUS MEDIUM

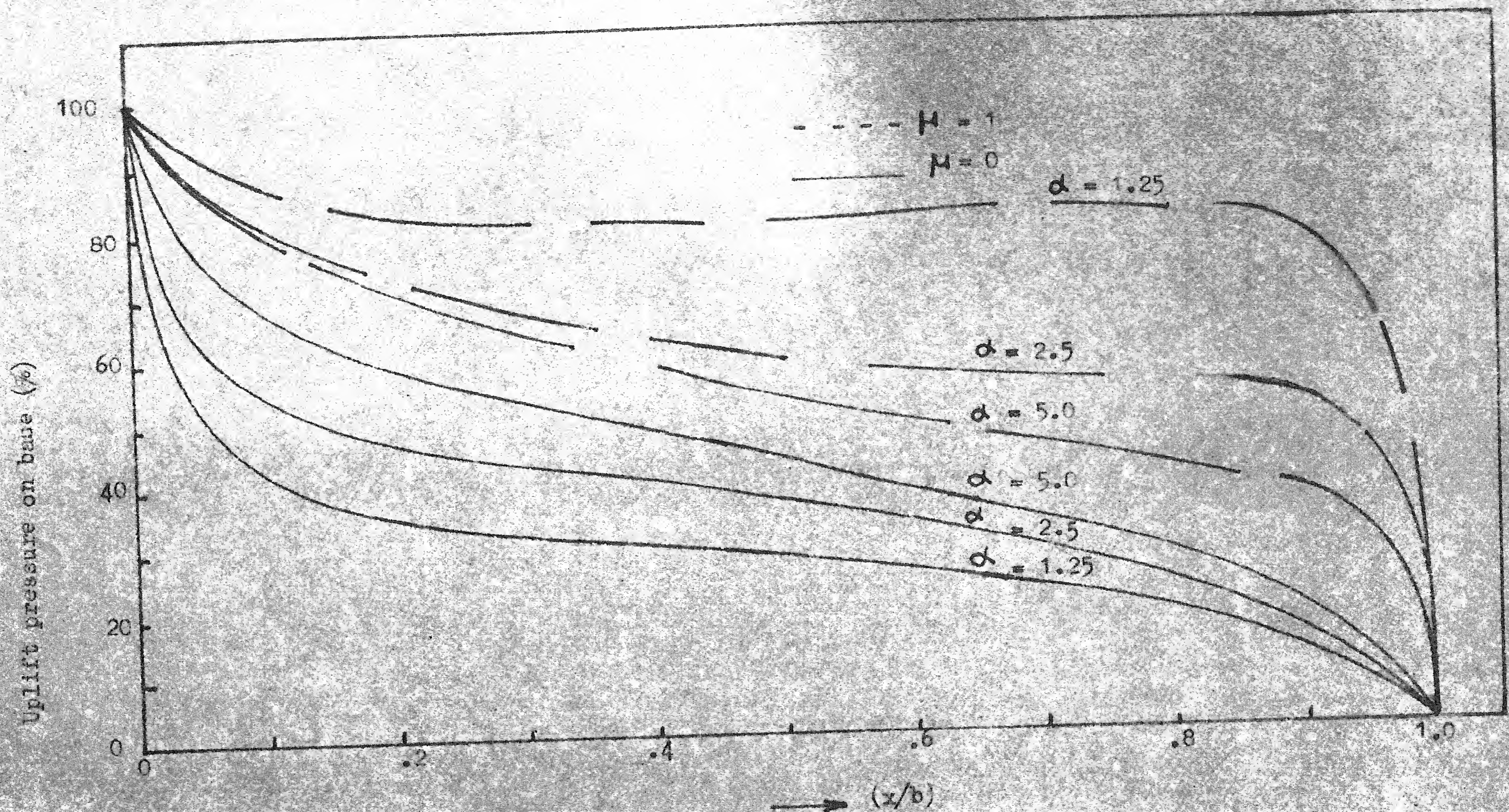


Fig. 5.7 Variation of uplift pressure along the base for $\theta = 60^{\circ}$ and $\gamma = 30^{\circ}$

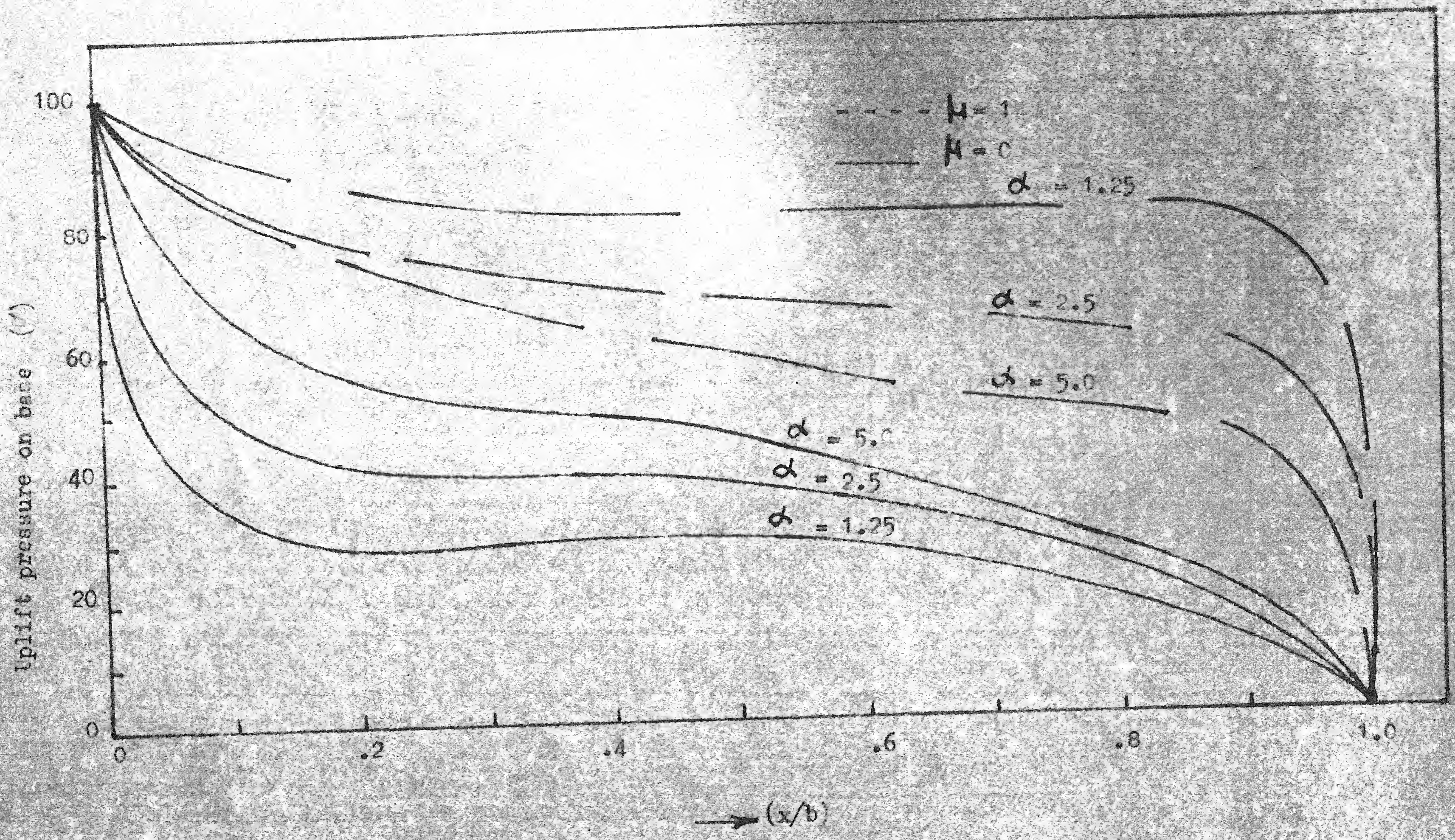


Fig. 5.8 Variation of the Uplift pressure for $\theta = 60^{\circ}$ and $\gamma = 60^{\circ}$

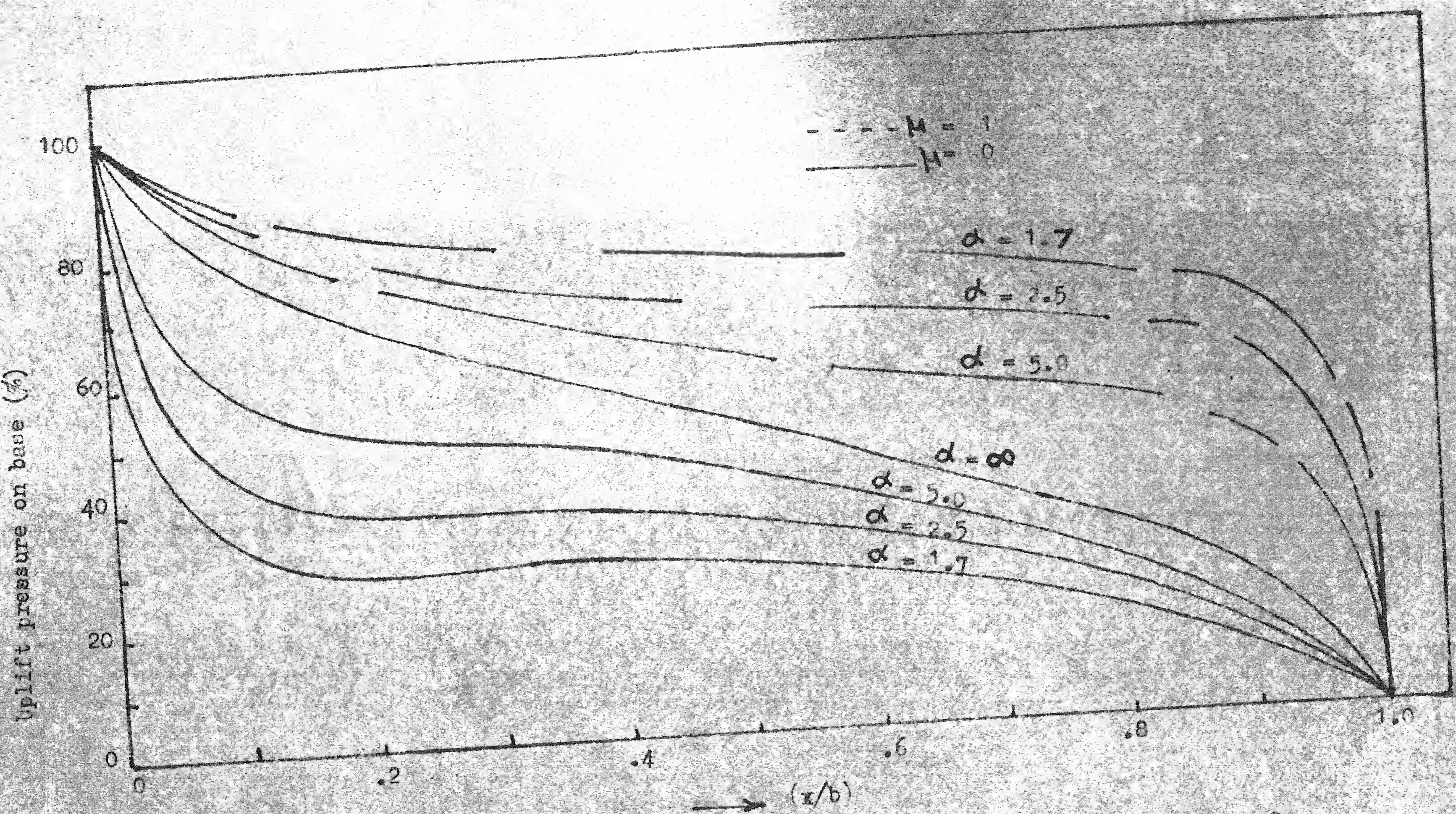


Fig. 5.9 Variation of Uplift pressure along the base for $\theta = 60^{\circ}, \gamma = 90^{\circ}$

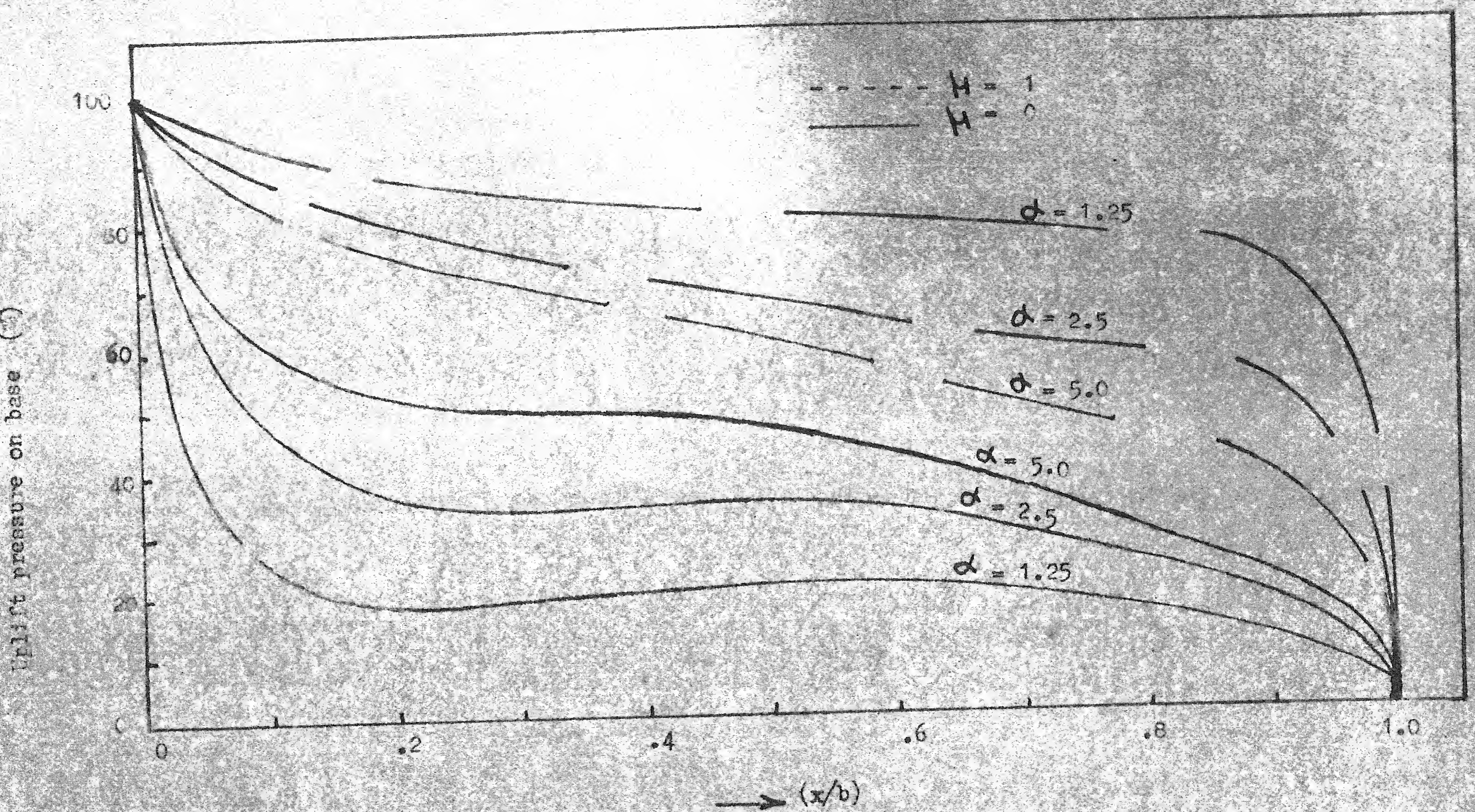


Fig. 5.10 Variation of uplift pressure along the base for $\theta = 60^{\circ}$, $\gamma = 120^{\circ}$

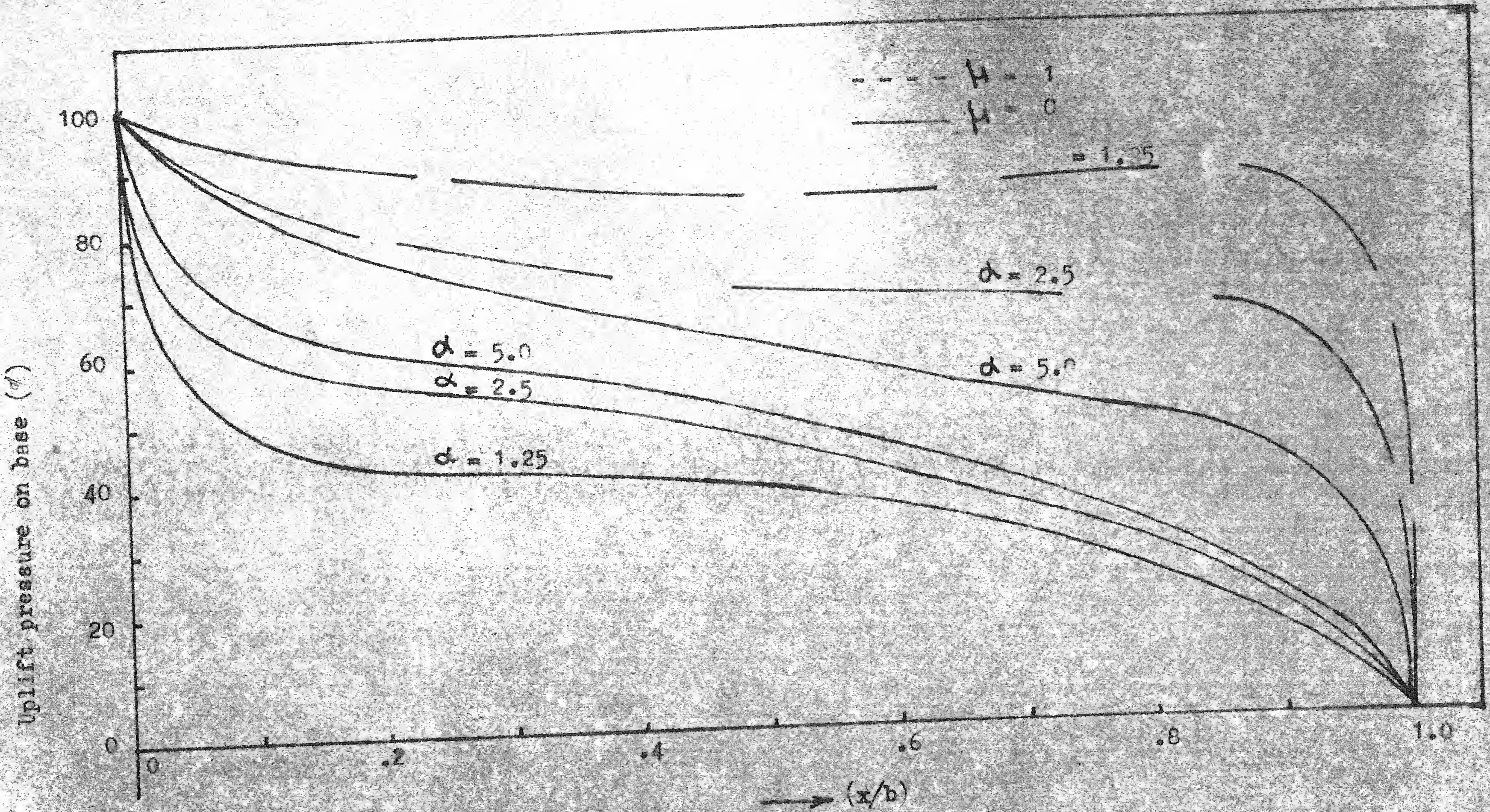


Fig. 5.11 Variation of uplift pressure along the base for $\theta = 30^{\circ}$, $\gamma = 30^{\circ}$

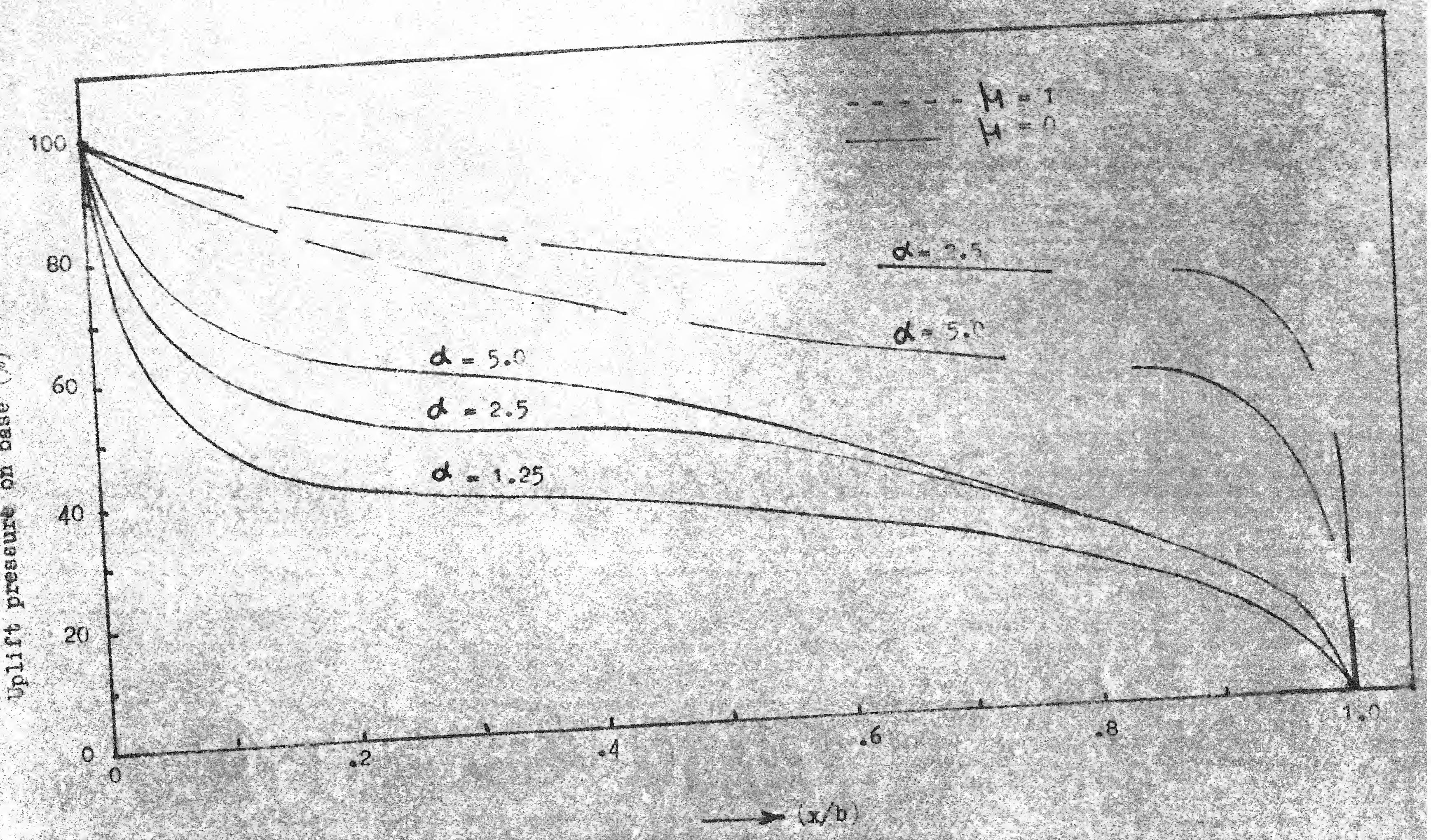


Fig. 5.12 Variation of uplift pressure along the base for $\theta = 30^{\circ}$, $\gamma = 60^{\circ}$

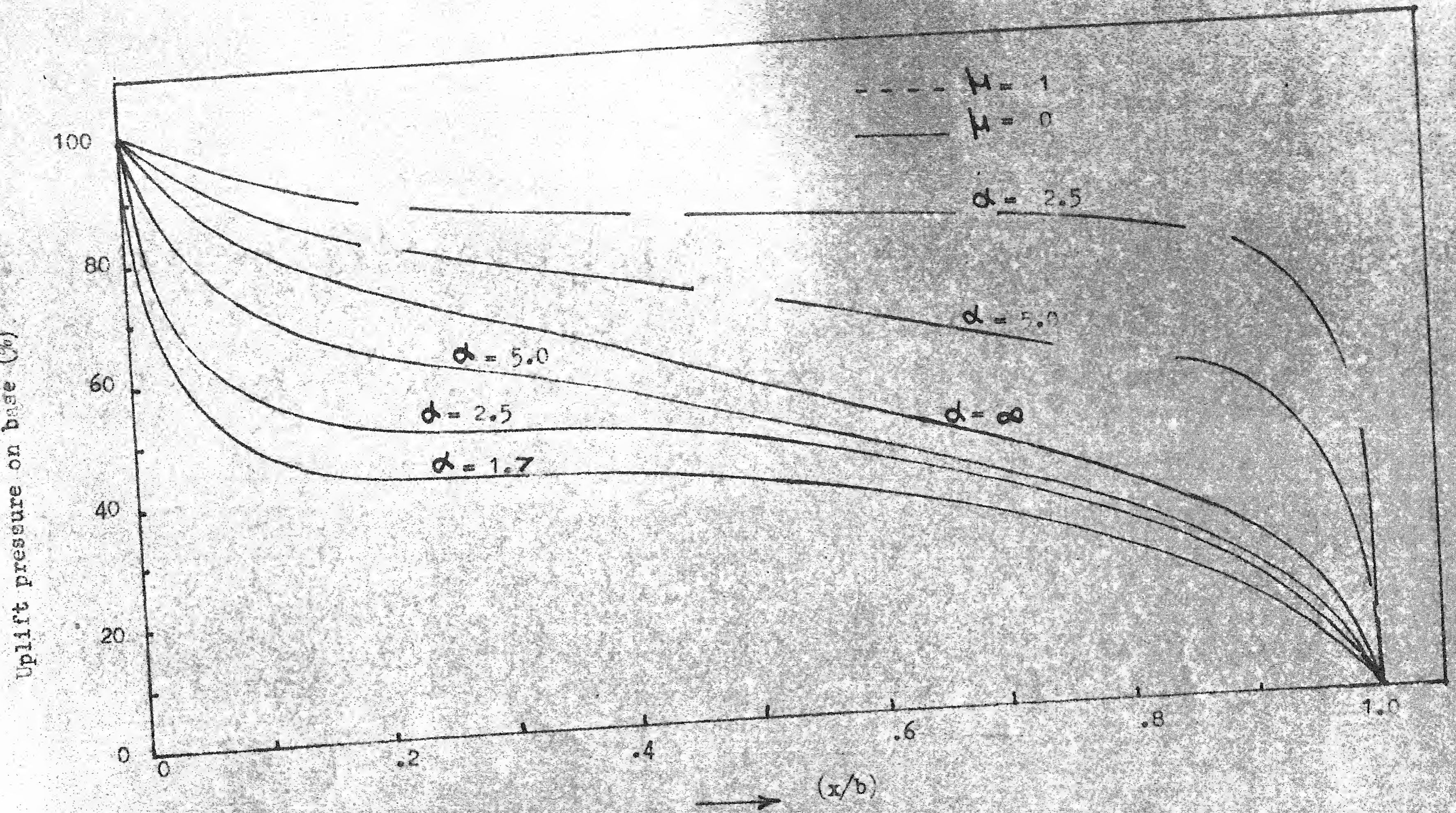


Fig. 5.13 Variation of uplift pressure along the base for $\theta = 30°$, $\gamma = 90°$

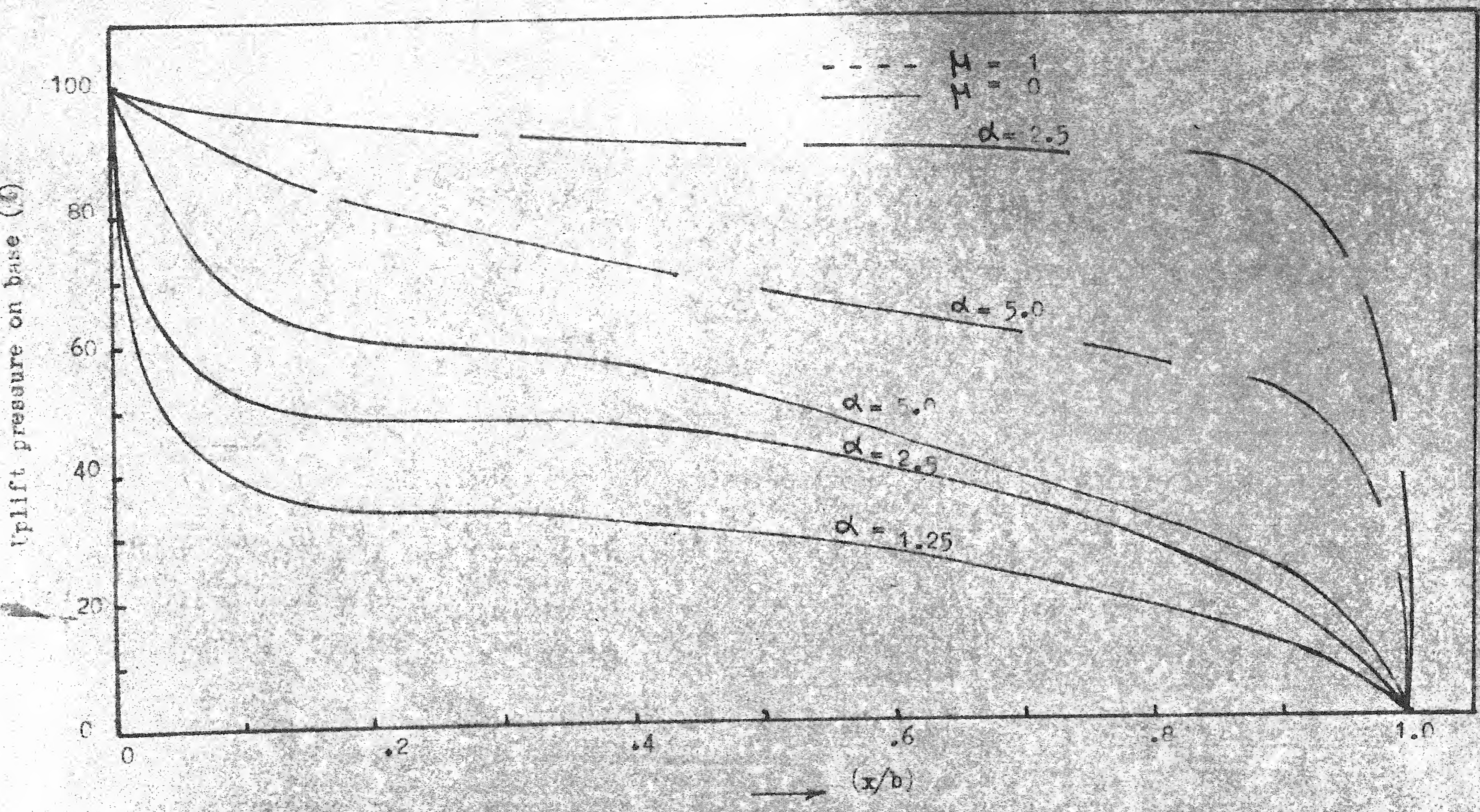


Fig. 5.14 Variation of uplift pressure along the base for θ = 30°, γ = 12°

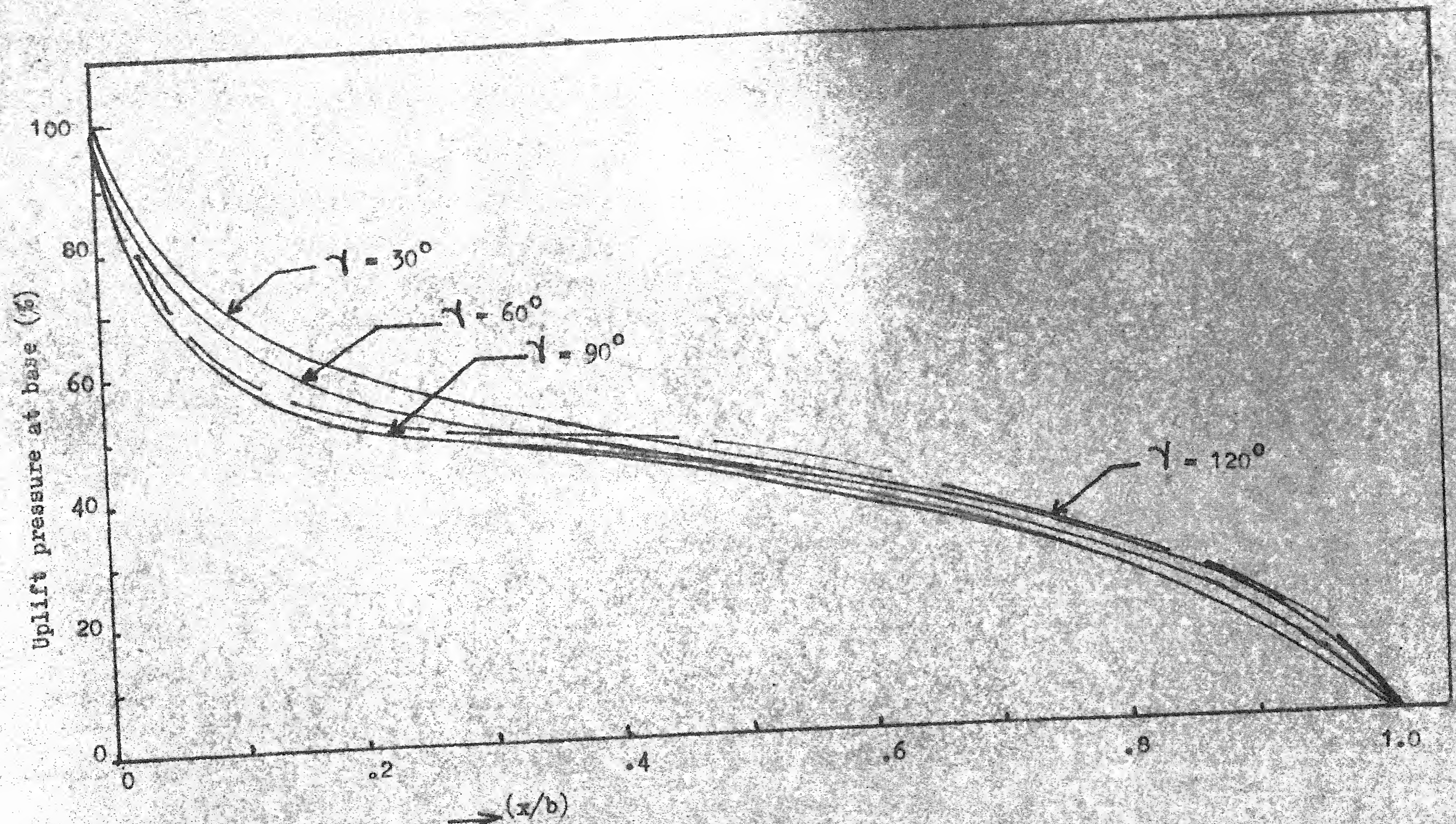


Fig. 5.15(a) Effect of on uplift pressure distribution at the base for $\mu = 0$, $\theta = 60°$ and $\alpha = 5.0$

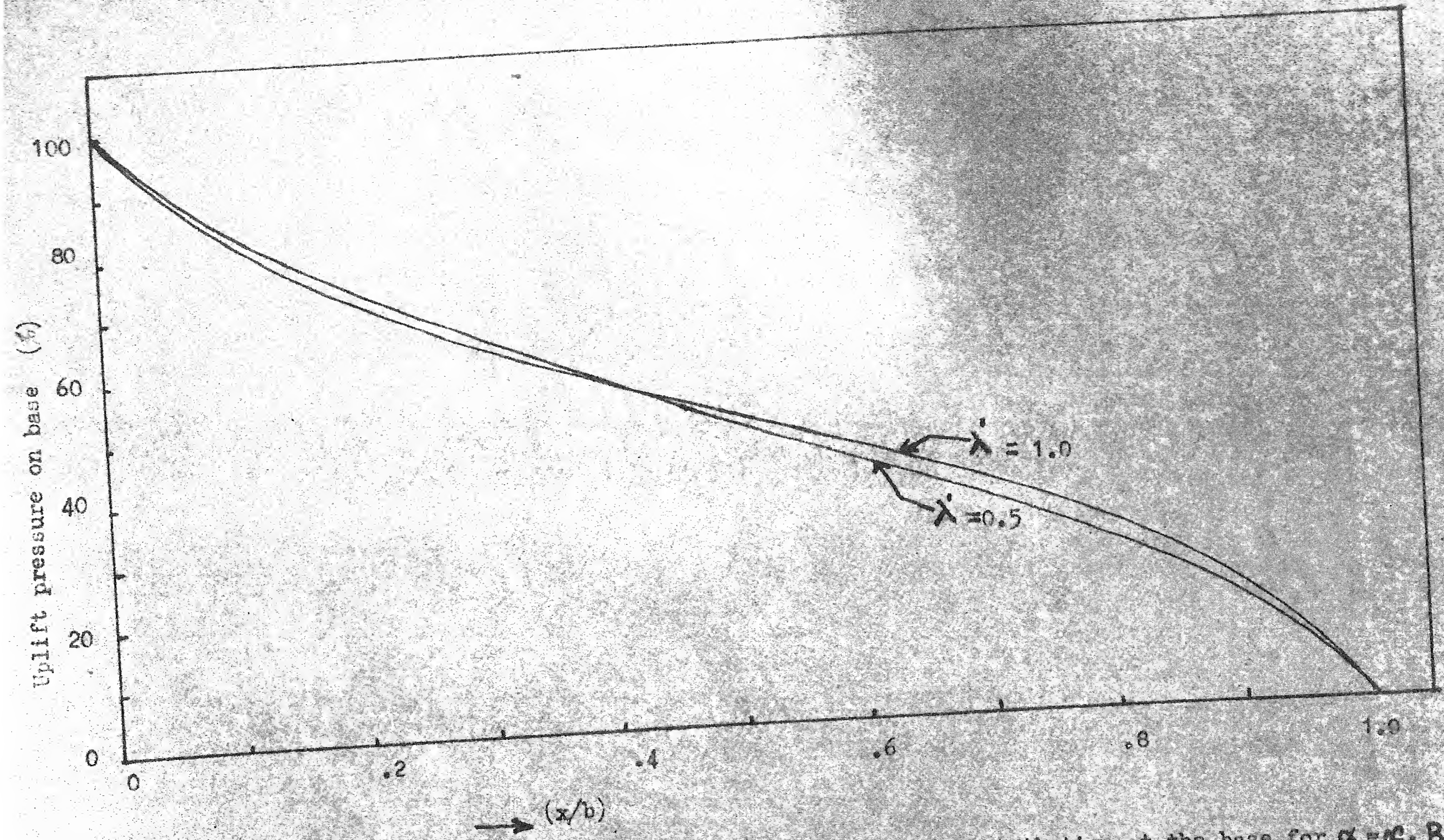


Fig. 5.15(b) Effect of λ' on uplift pressure distribution at the base for $\alpha = \infty$, $\beta = 1.0$ and $\theta = 60^{\circ}$

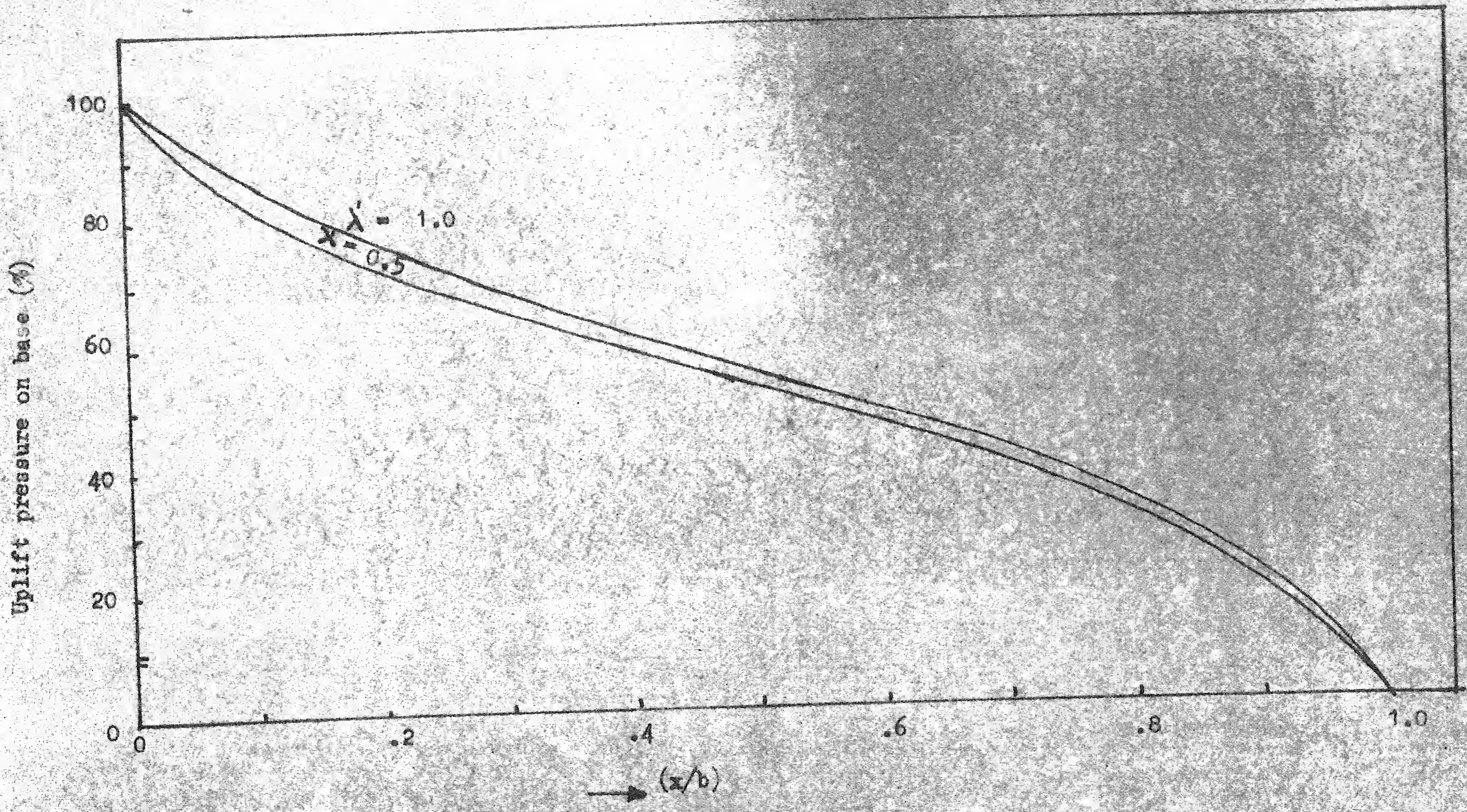


Fig. 5.15 (c) Effect of λ' on uplift pressure distribution at the base for $\alpha = \alpha$, $\beta = 1.0$ and $\theta = 30^0$

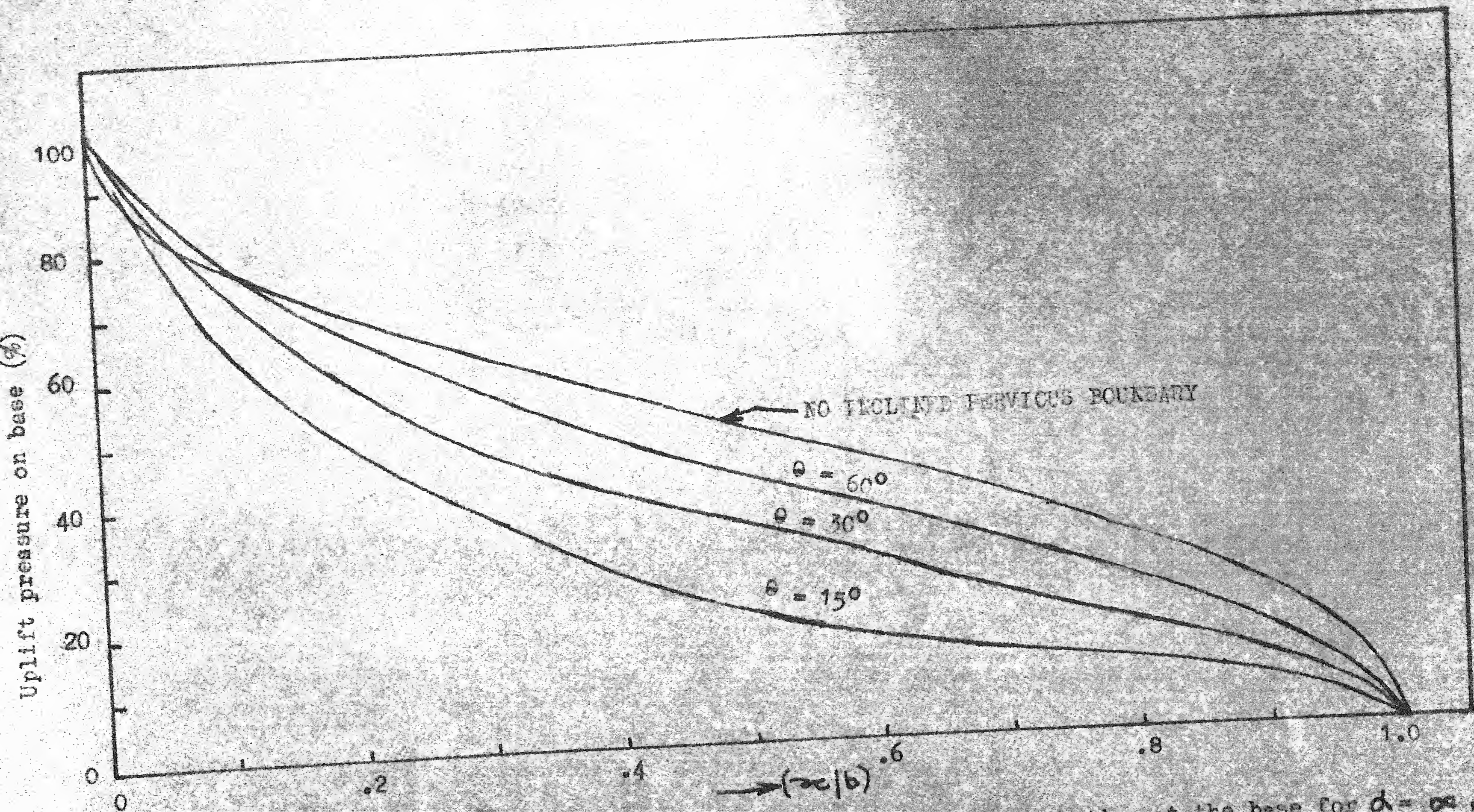


Fig. 5.16 Effect of θ on uplift pressure distribution at the base for $\alpha = \infty$, $\beta = 1.65$

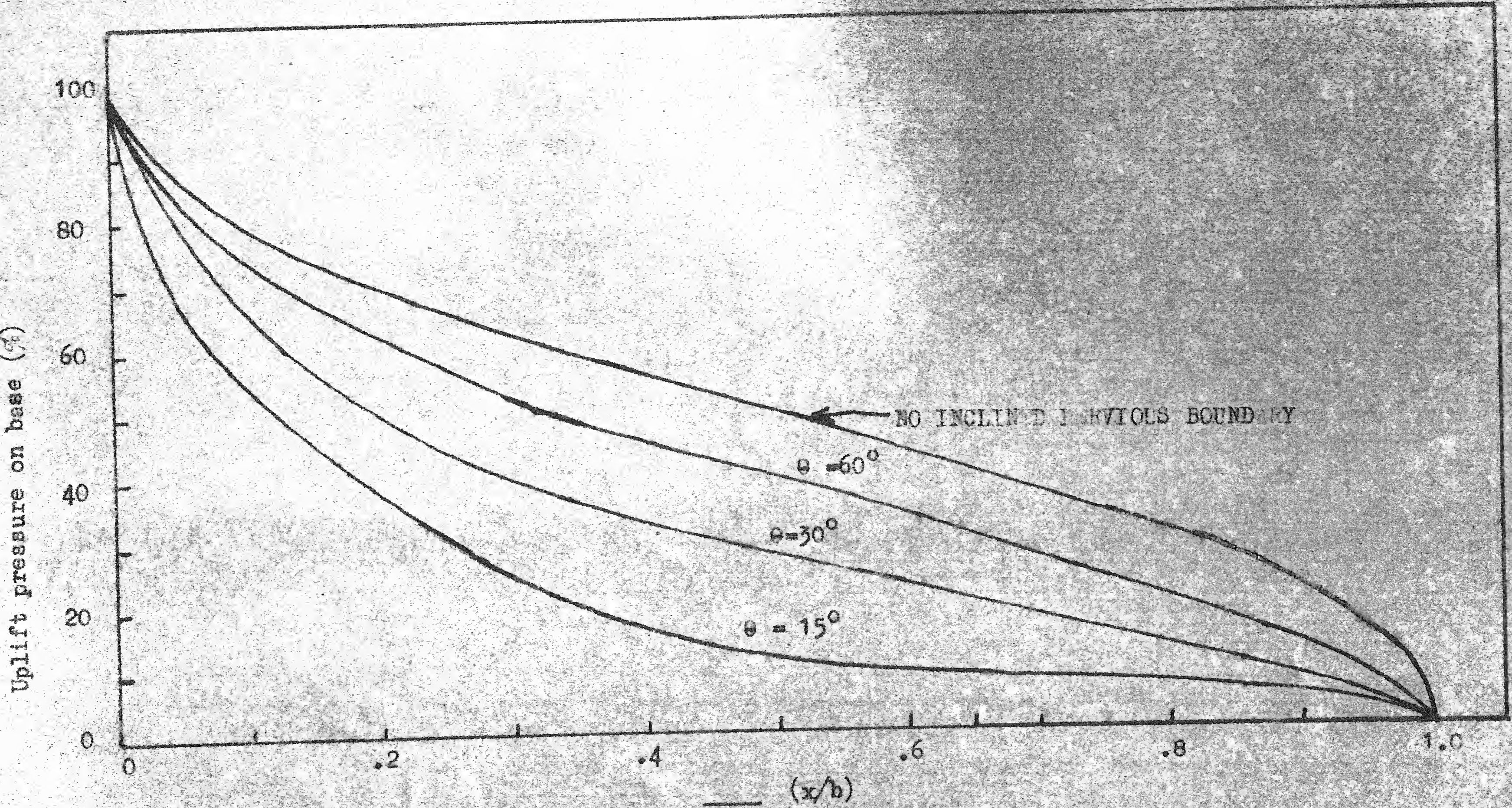


Fig. 5.17 Effect of θ on uplift pressure distribution at the base for $\alpha = \infty$, $\beta = 1.0$

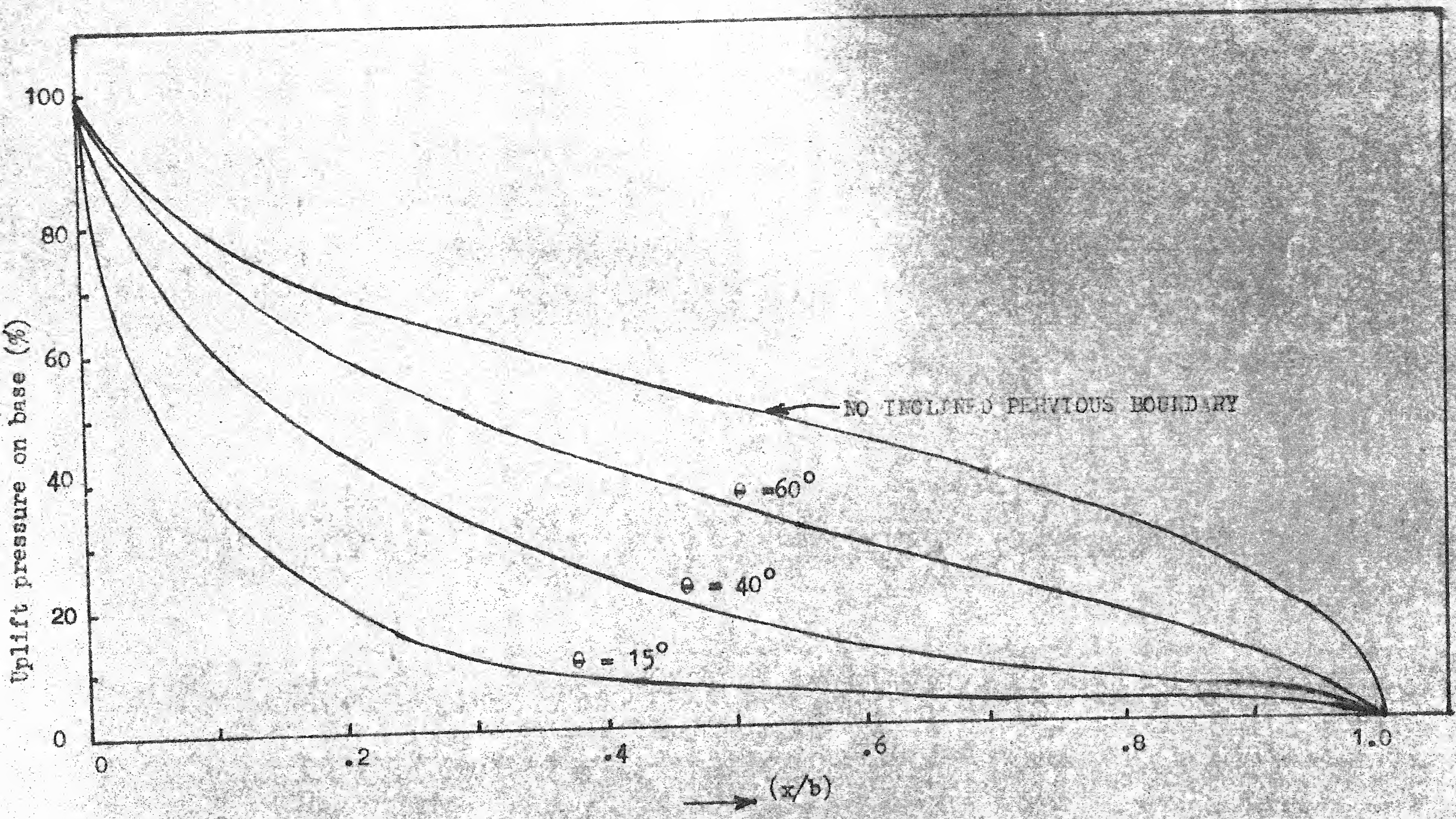


Fig. 5.18 Effect of θ on uplift pressure distribution at the base for $\alpha = \infty$, $\beta = 0.5$.

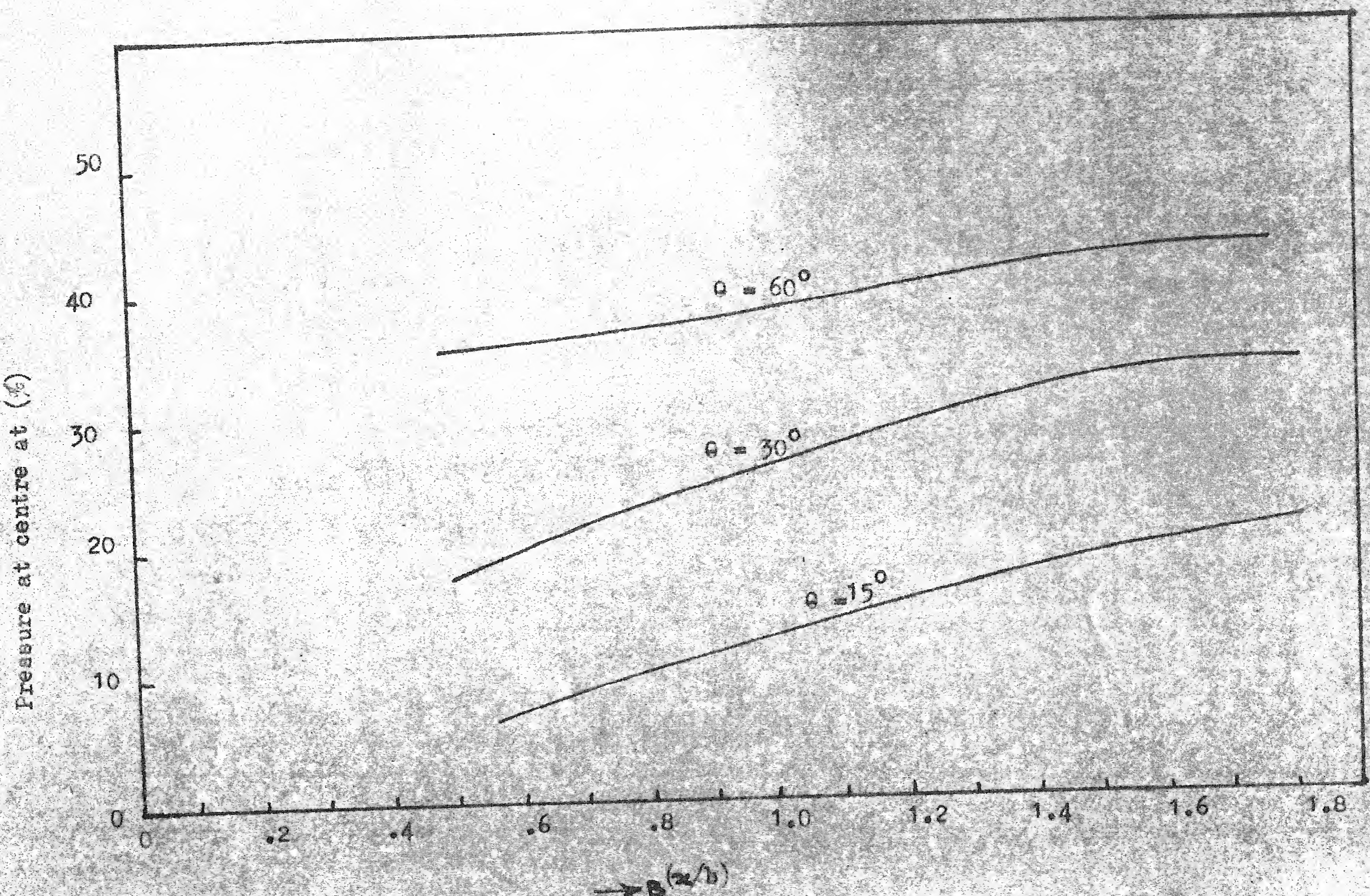


Fig. 5.19 Effect of θ and β on pressure at the centre of the weir.

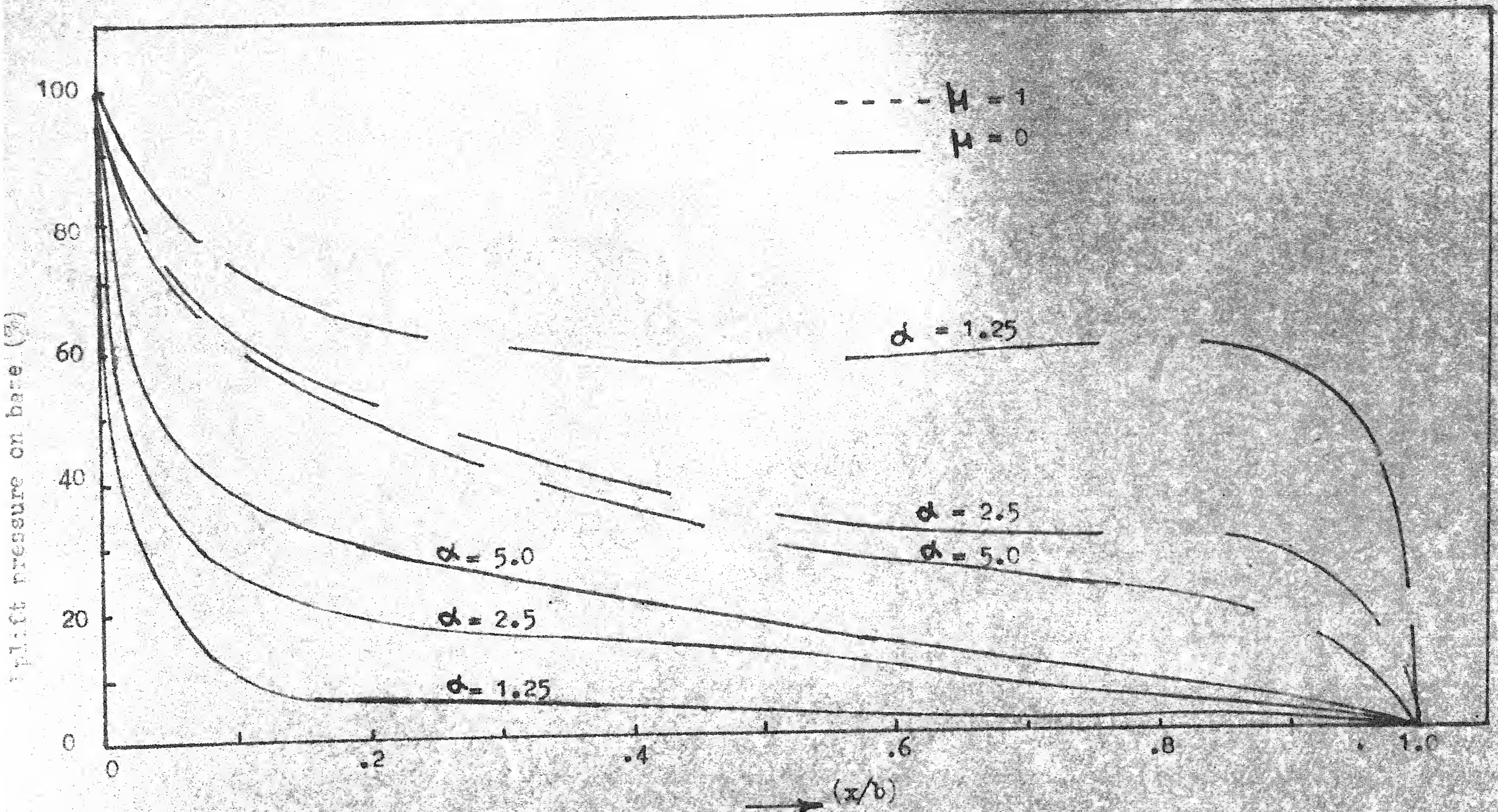


Fig. 5.20 Variation of uplift pressure distribution at the base for $\theta = 30^{\circ}$, $\gamma = 30^{\circ}$

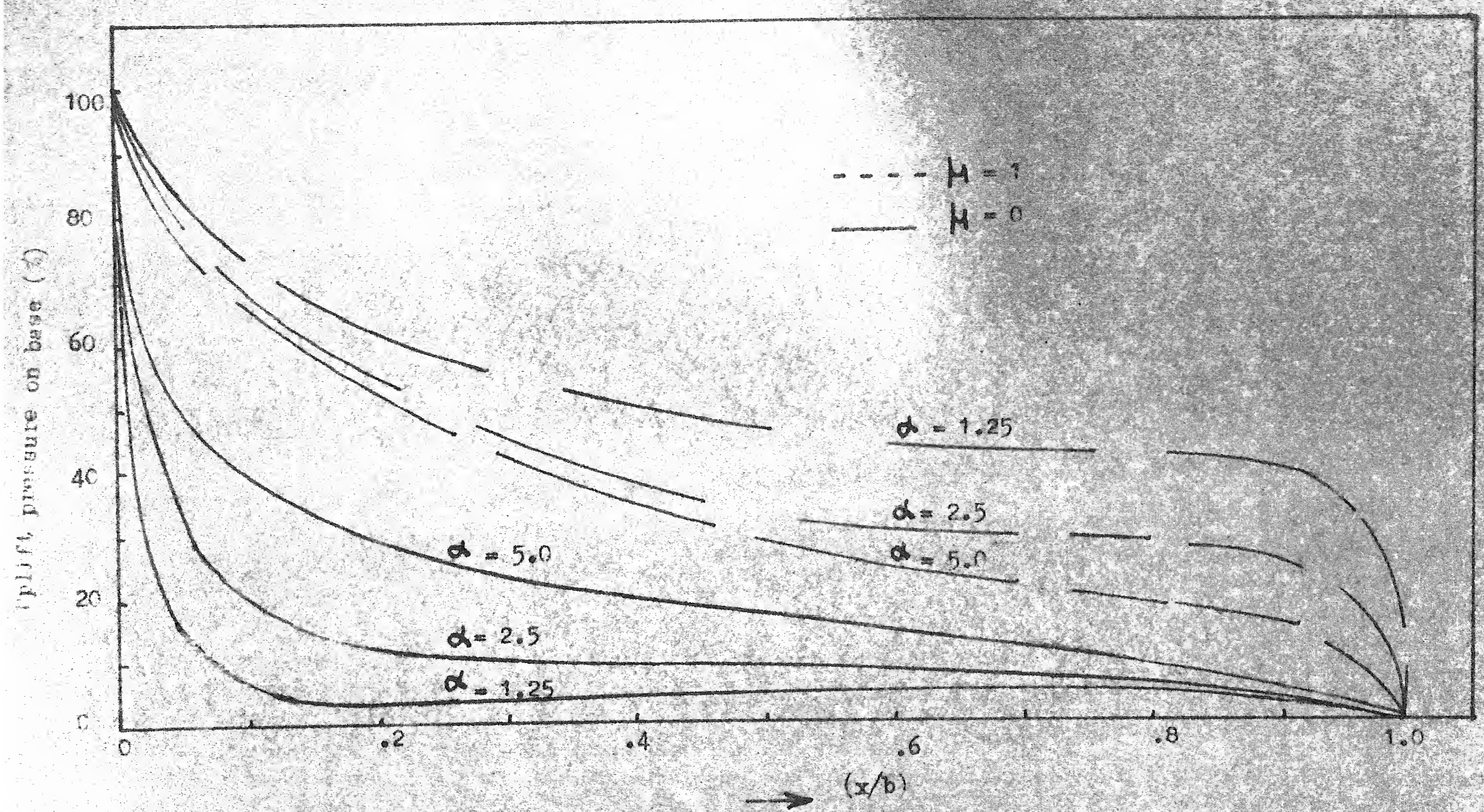


Fig. 5.21 Variation of uplift pressure distribution at the base for $\theta = 30^{\circ}$, $\gamma = 60^{\circ}$

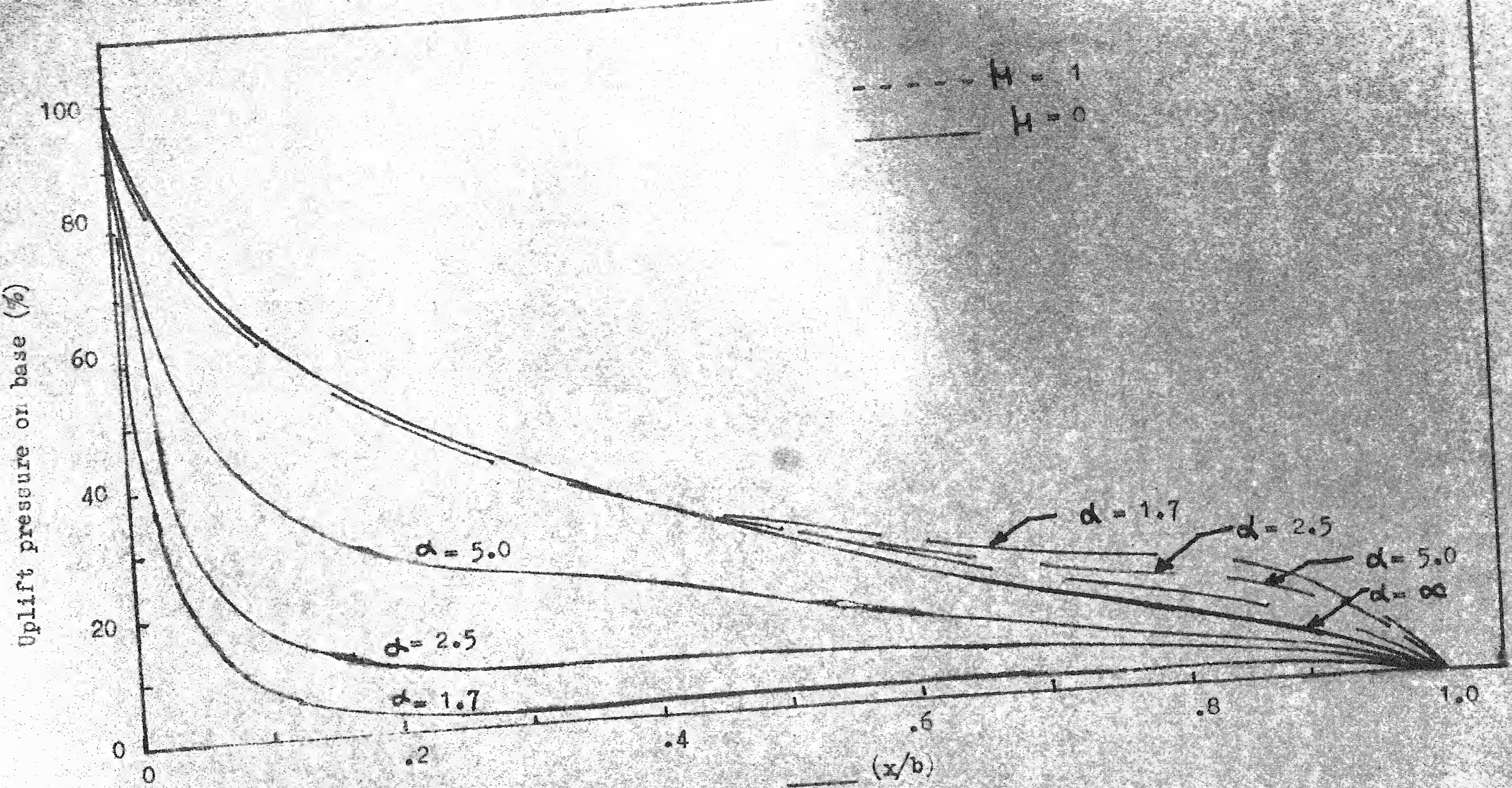


Fig. 5.22 Variation of uplift pressure distribution at the base for $\theta = 30^{\circ}, \gamma = 90^{\circ}$

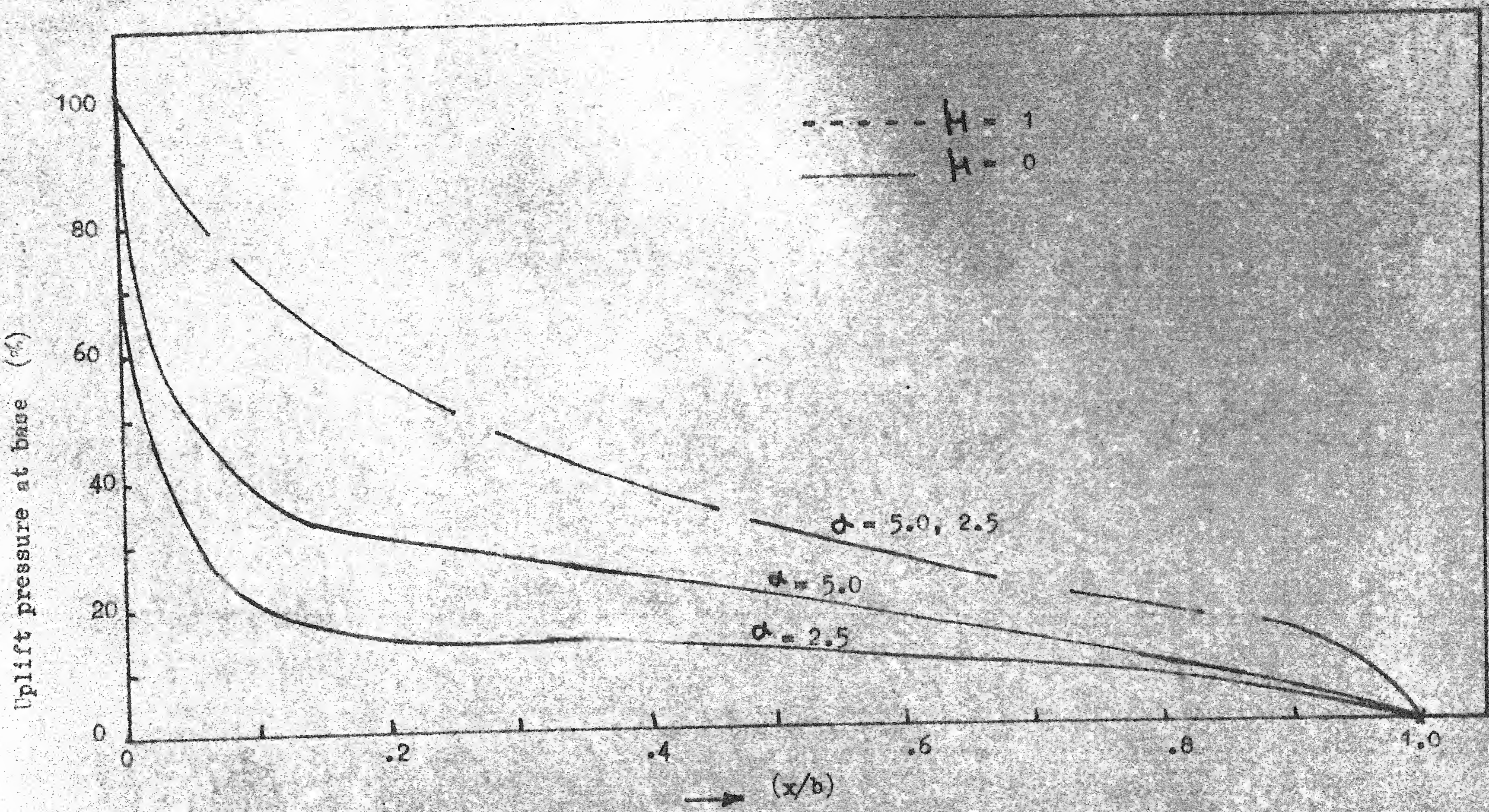


Fig. 5.23 Variation of uplift pressure distribution at the base for $\theta = 30^{\circ}, \gamma = 120^{\circ}$

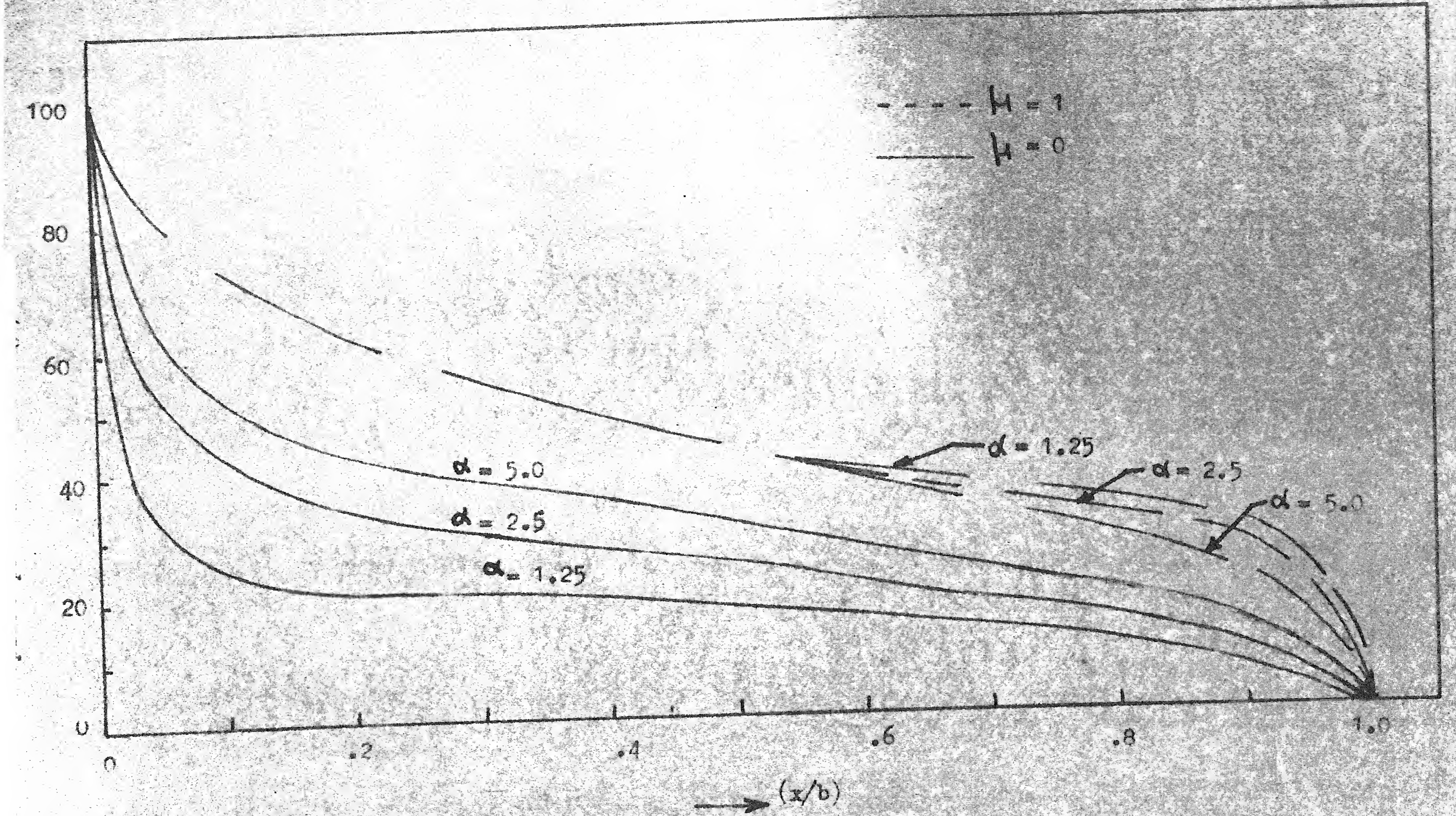


Fig. 5.24 Variation of uplift pressure distribution at the base for $\theta = 60^{\circ}$, $\gamma = 30^{\circ}$

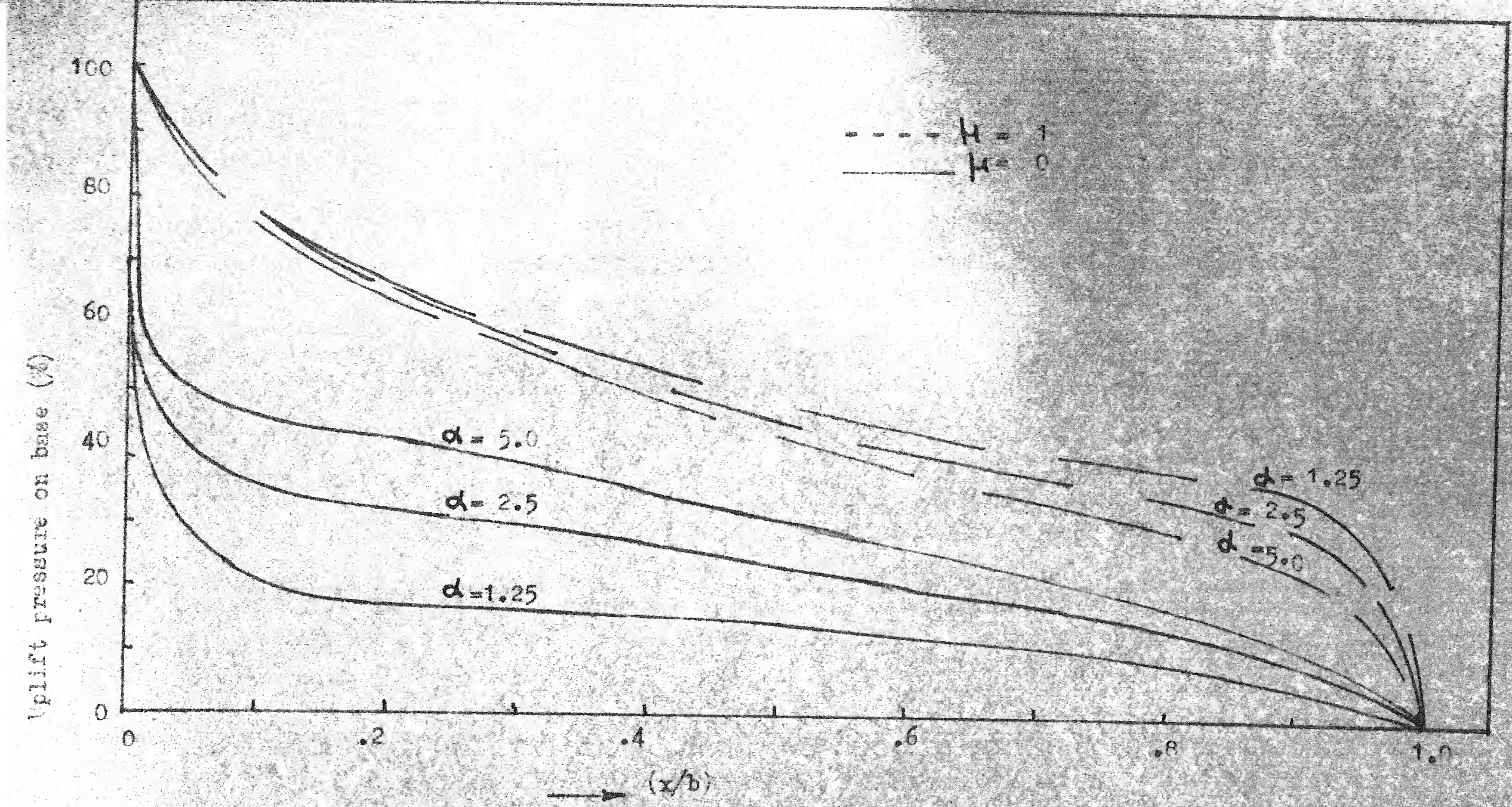


Fig. 5.25 Variation of uplift pressure distribution at the base for $\theta = 60^\circ$, $\gamma = 60^\circ$

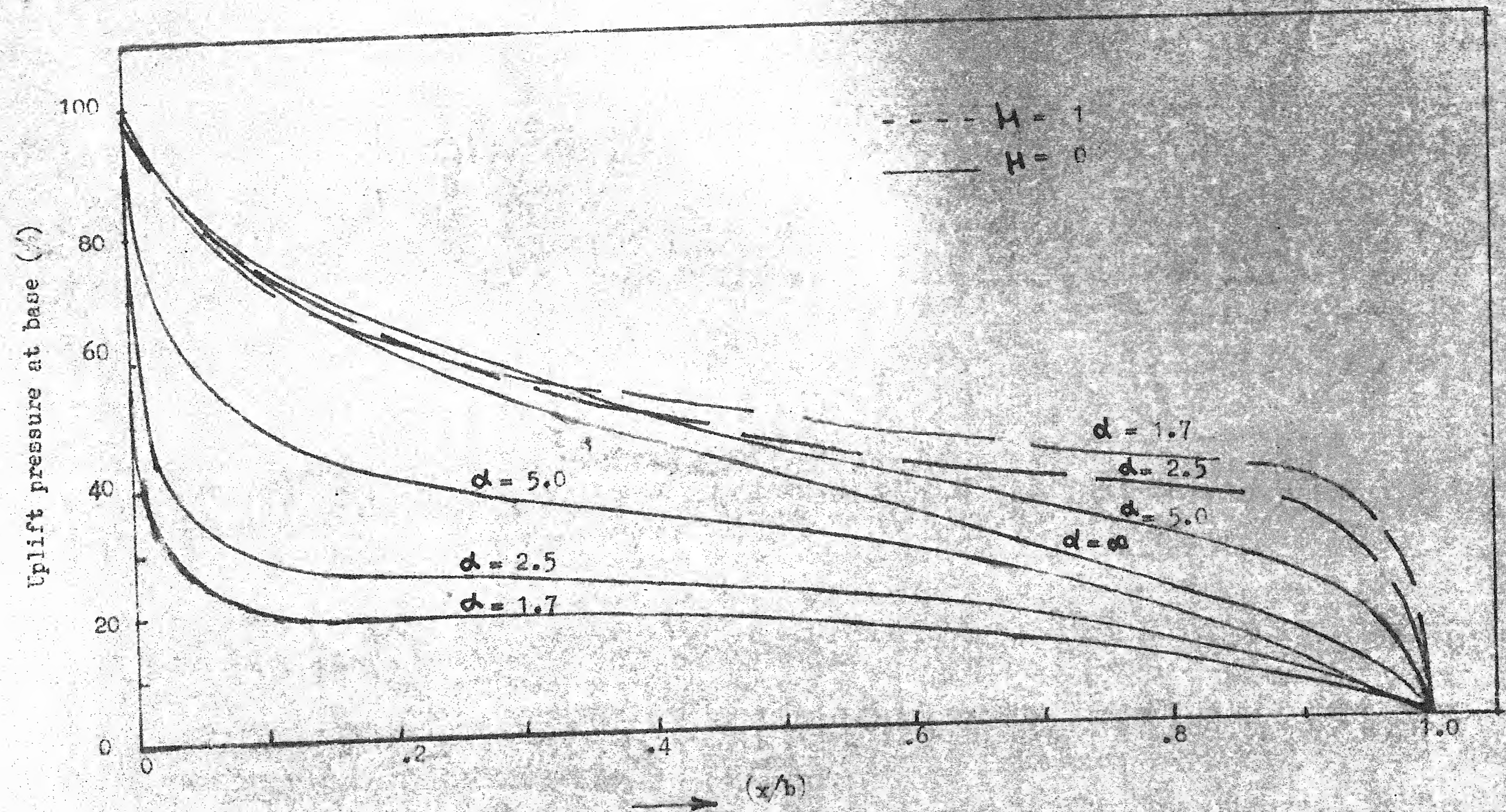


Fig. 5.26 Variation of uplift pressure distribution at the base for $\theta = 60^{\circ}$, $\gamma = 90^{\circ}$

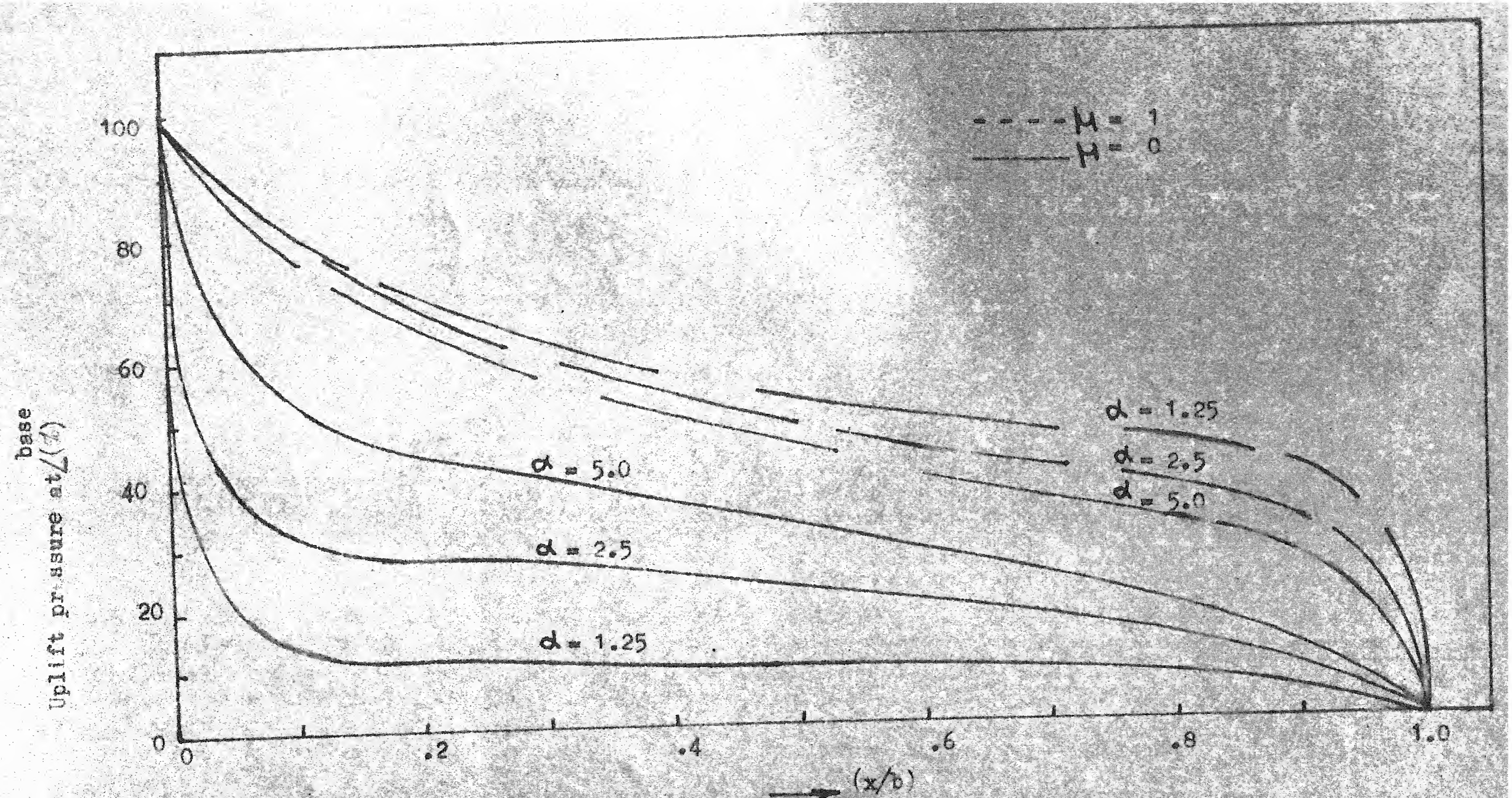


Fig. 5.27 Variation of uplift pressure distribution at the base for $\theta = 60^o$, $\gamma = 120^o$

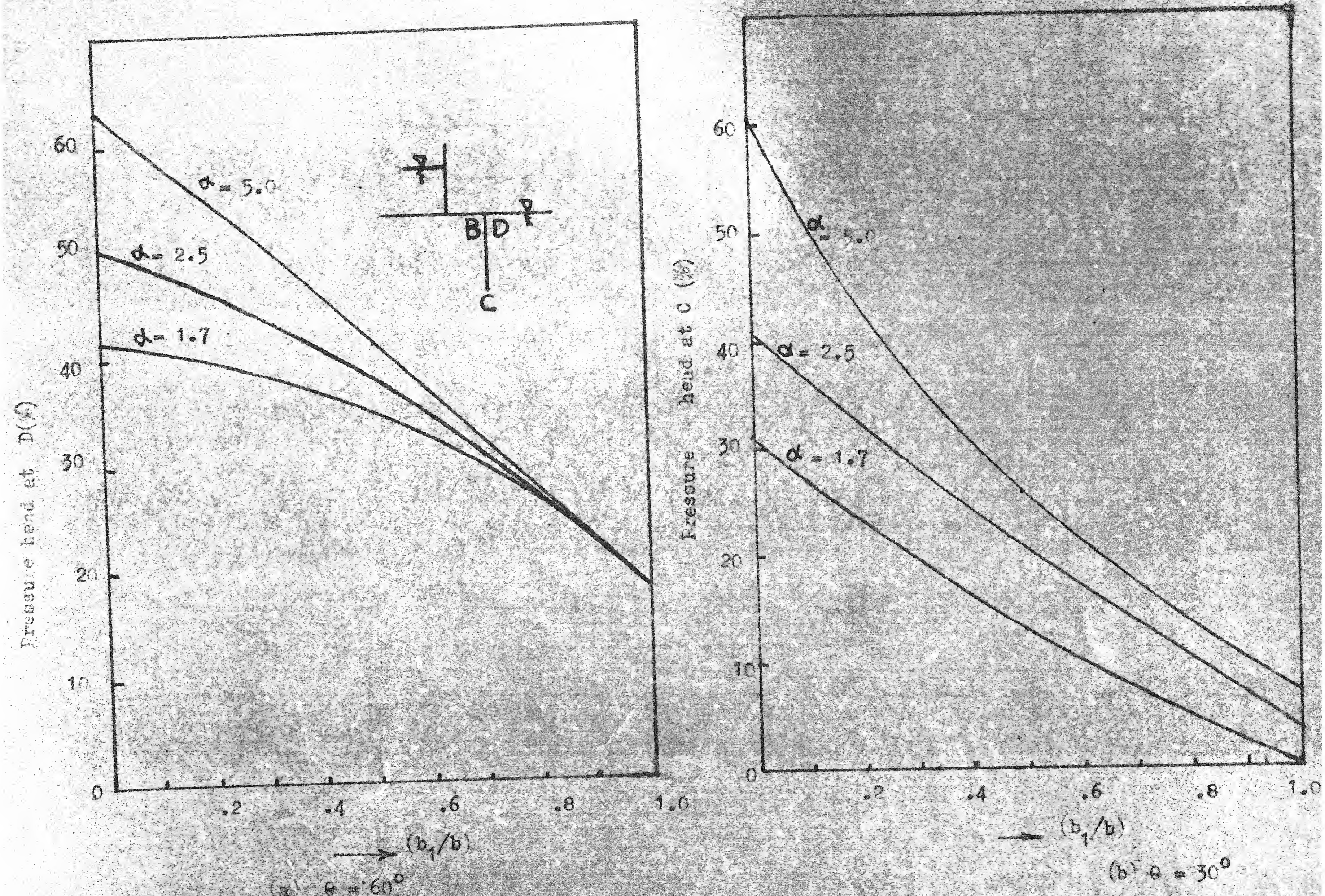
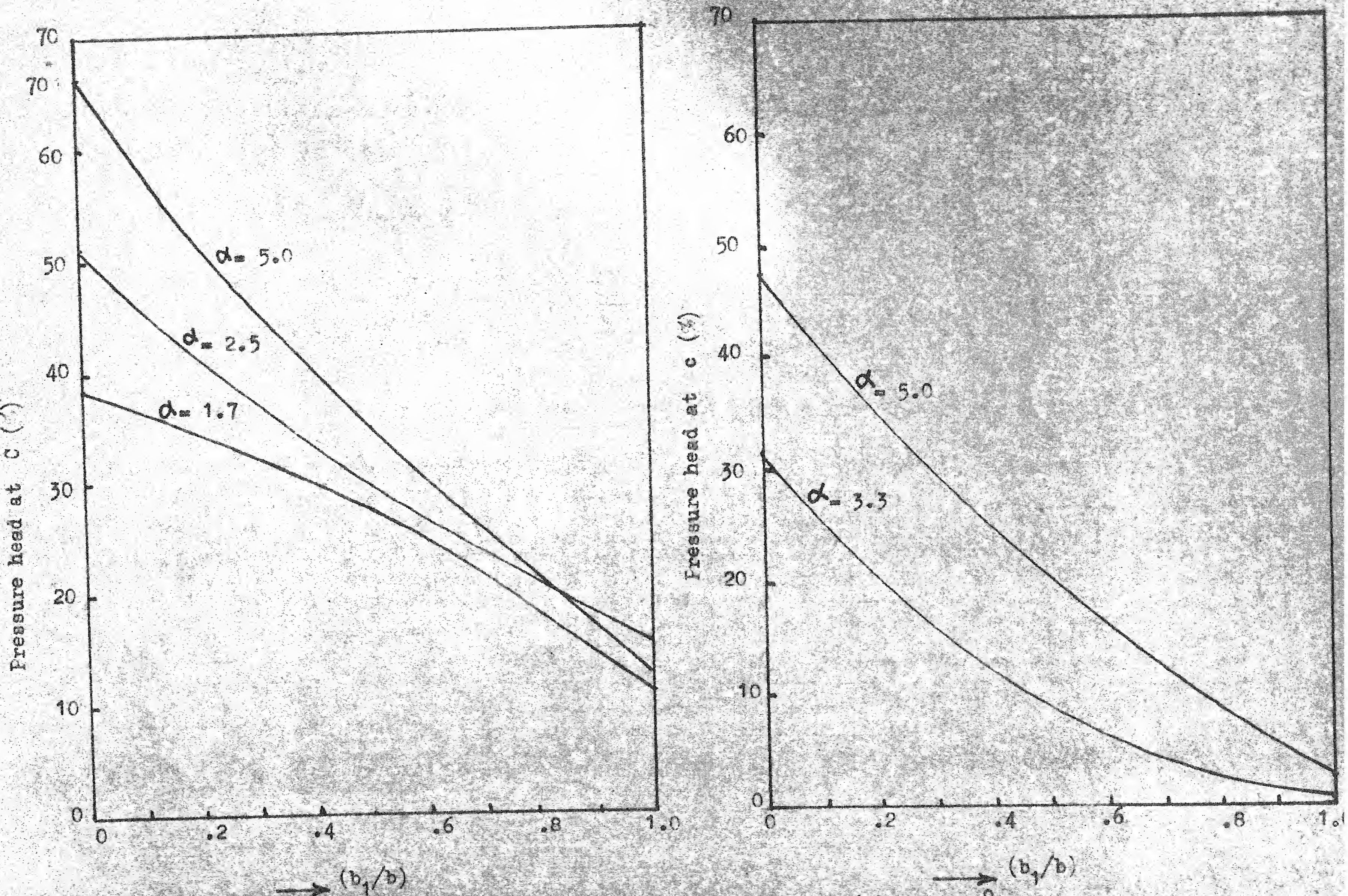


Fig. 5.28 Effect of position of sheet pile on pressure head at ... α = 60°, θ = 30° for γ = ... β



(a) $\theta = 60^{\circ}$

(b) $\theta = 30^{\circ}$

Fig. 5.29 Effect of position of sheet pile on pressure head at tip of the pile for $\theta = 60^{\circ}$ and $\theta =$ for $\gamma = 90^{\circ}$ and $\beta = .5$

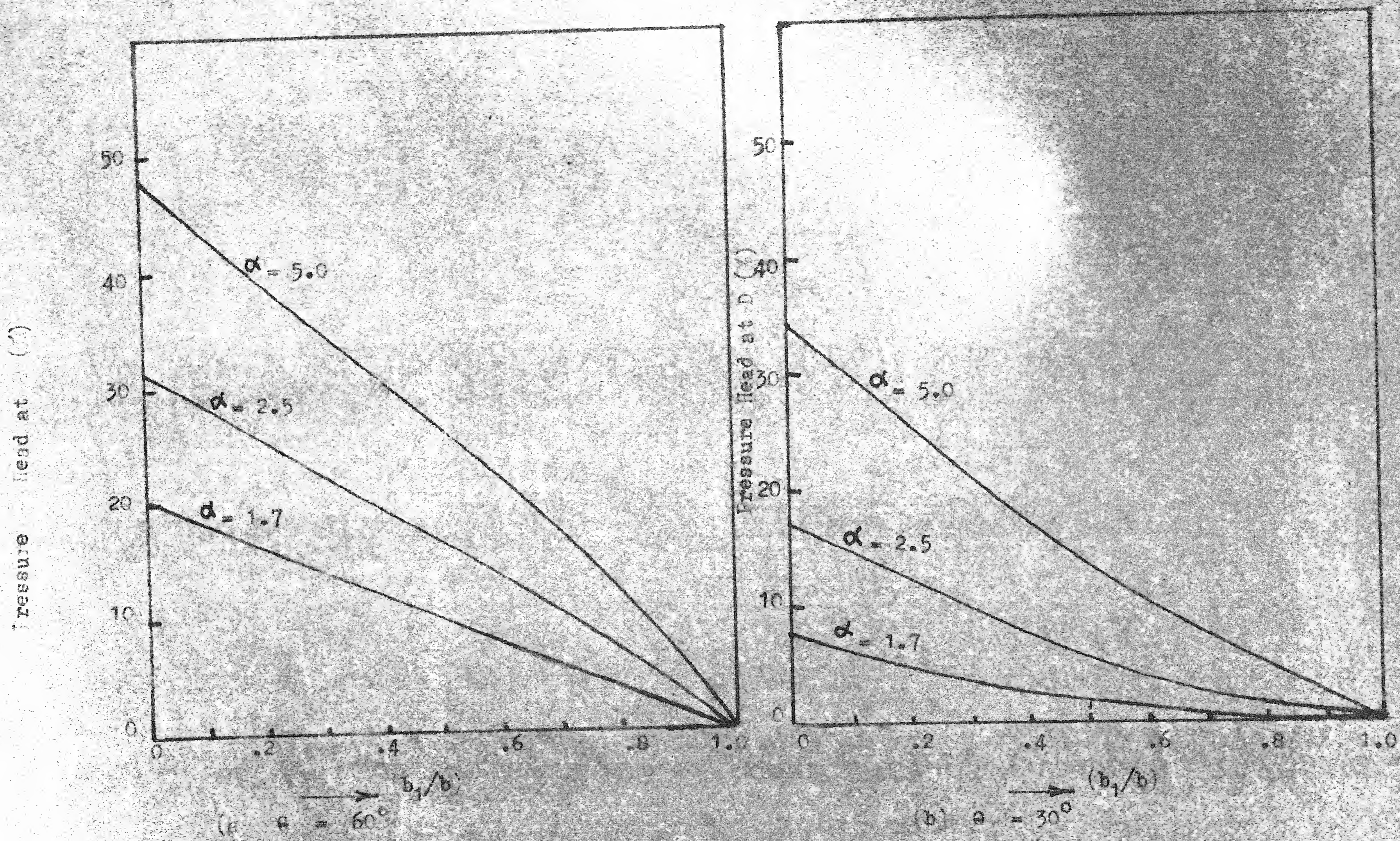


Fig. .30 Effect of position of sheet pile on pressure head at D for θ = 60° and 30° for γ = 90° and β = 1.0

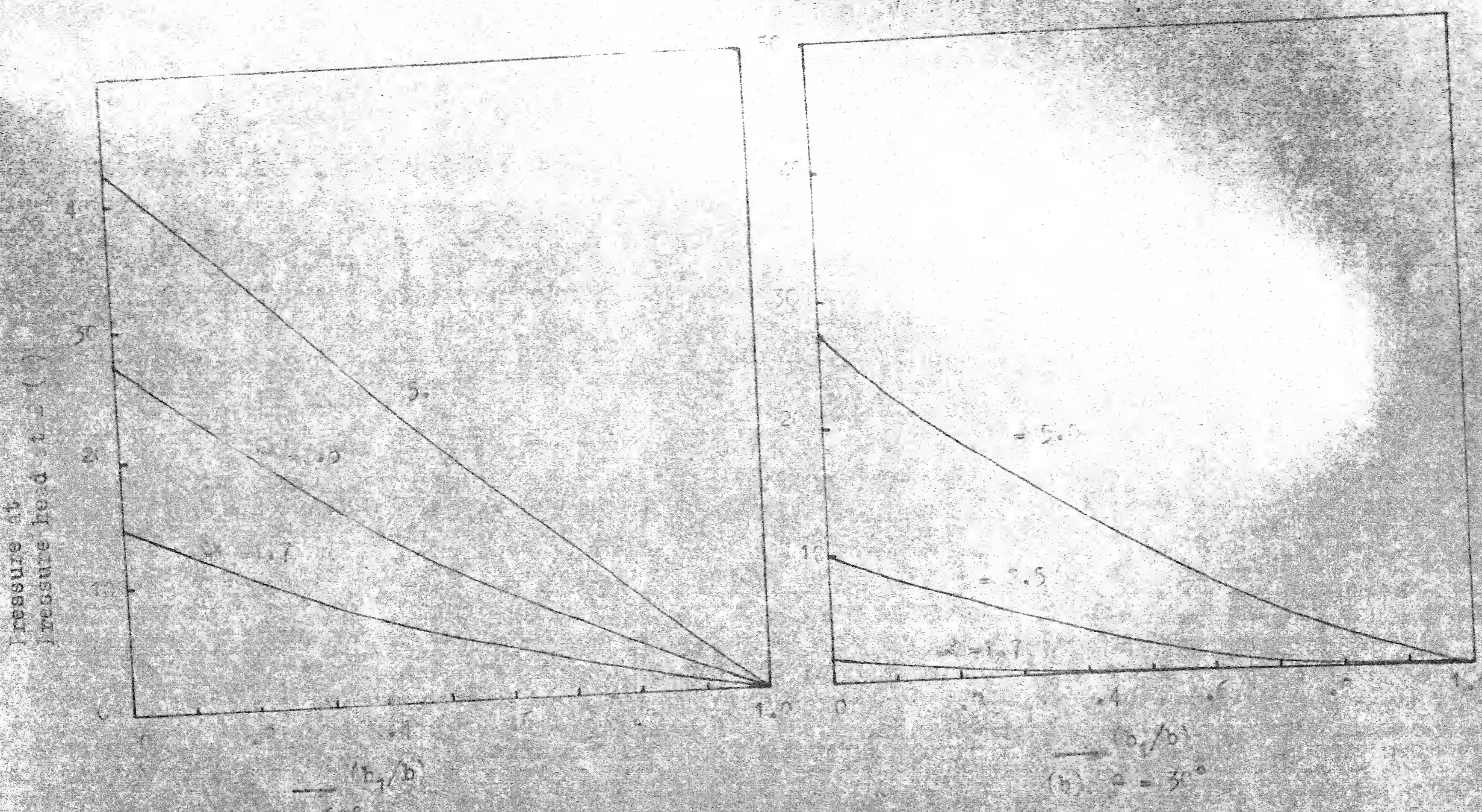


Fig. 5.31 Effect of position of sheet pile on pressure head at D for θ = 60° and θ = 30° for γ = 90° and β = .

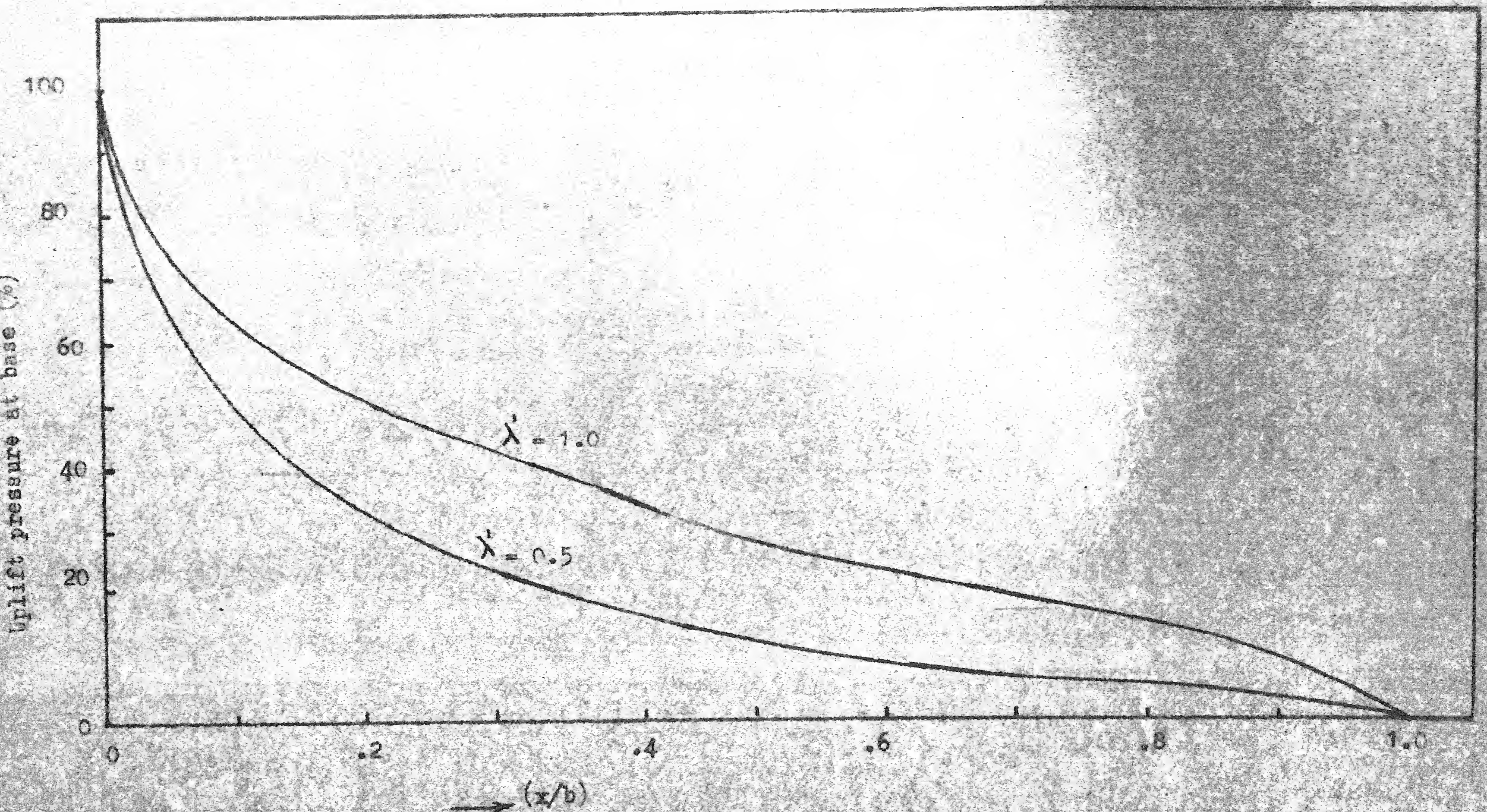


Fig. 5.32 Effect of λ' on uplift pressure distribution at the base for $\theta = 60^{\circ}$ and $\beta = 1.0$.

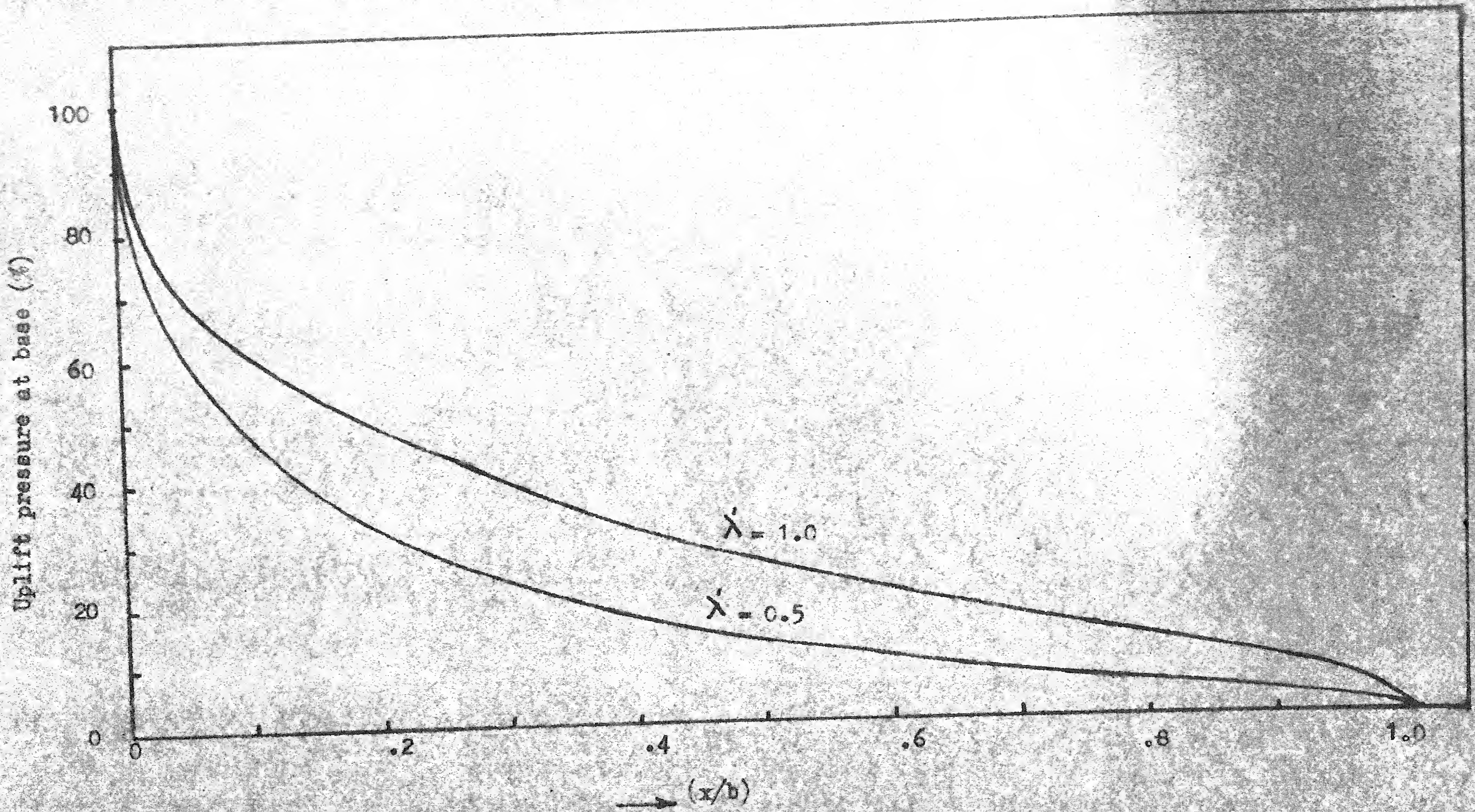


Fig. 5.33 Effect of λ' on uplift pressure distribution at the base for $\alpha = 30^0$, $\beta = .5$

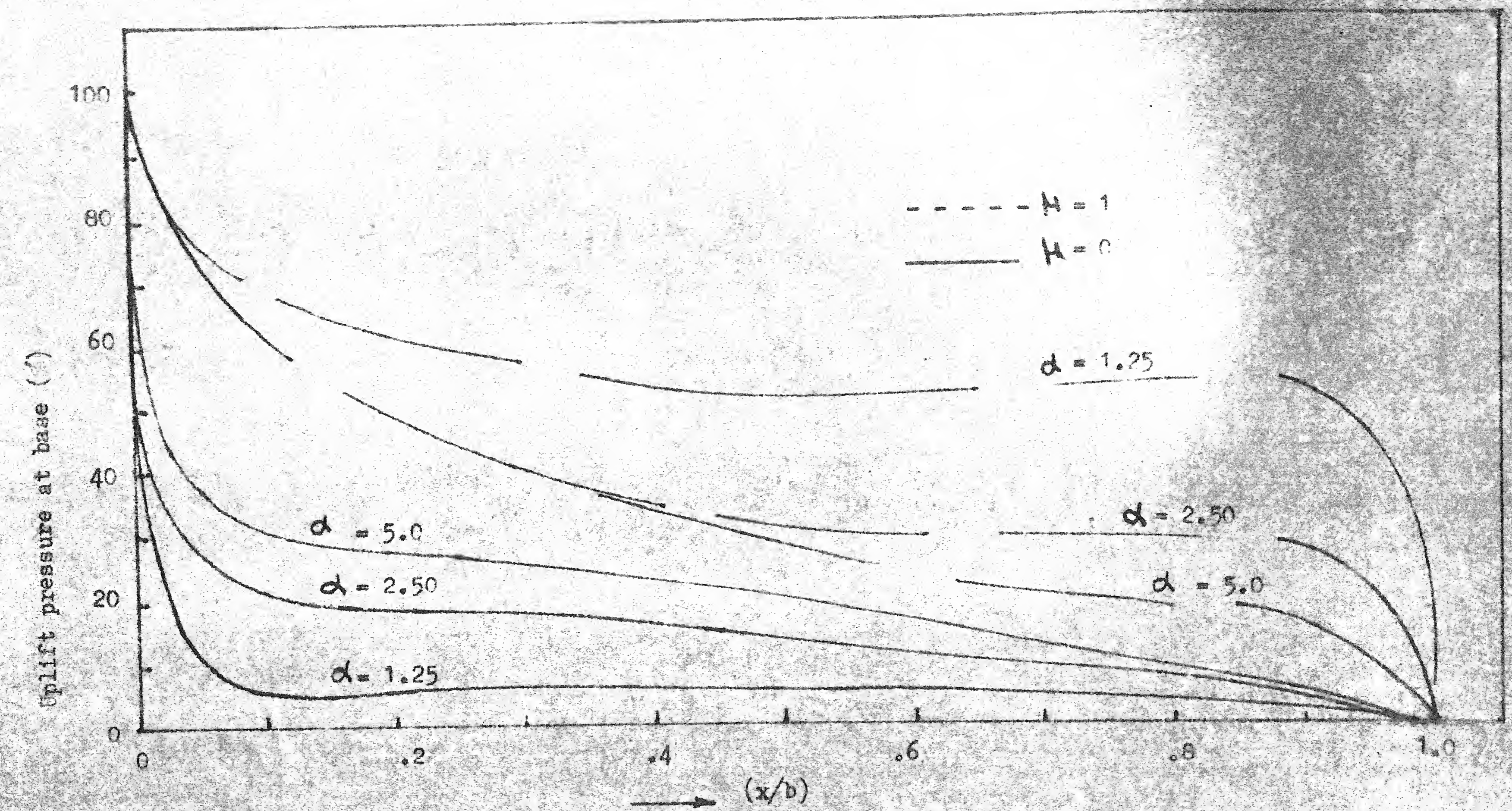


Fig. 5.34 Variation of uplift pressure at the base for $\theta = 60^{\circ}$ and $\gamma = 30^{\circ}$

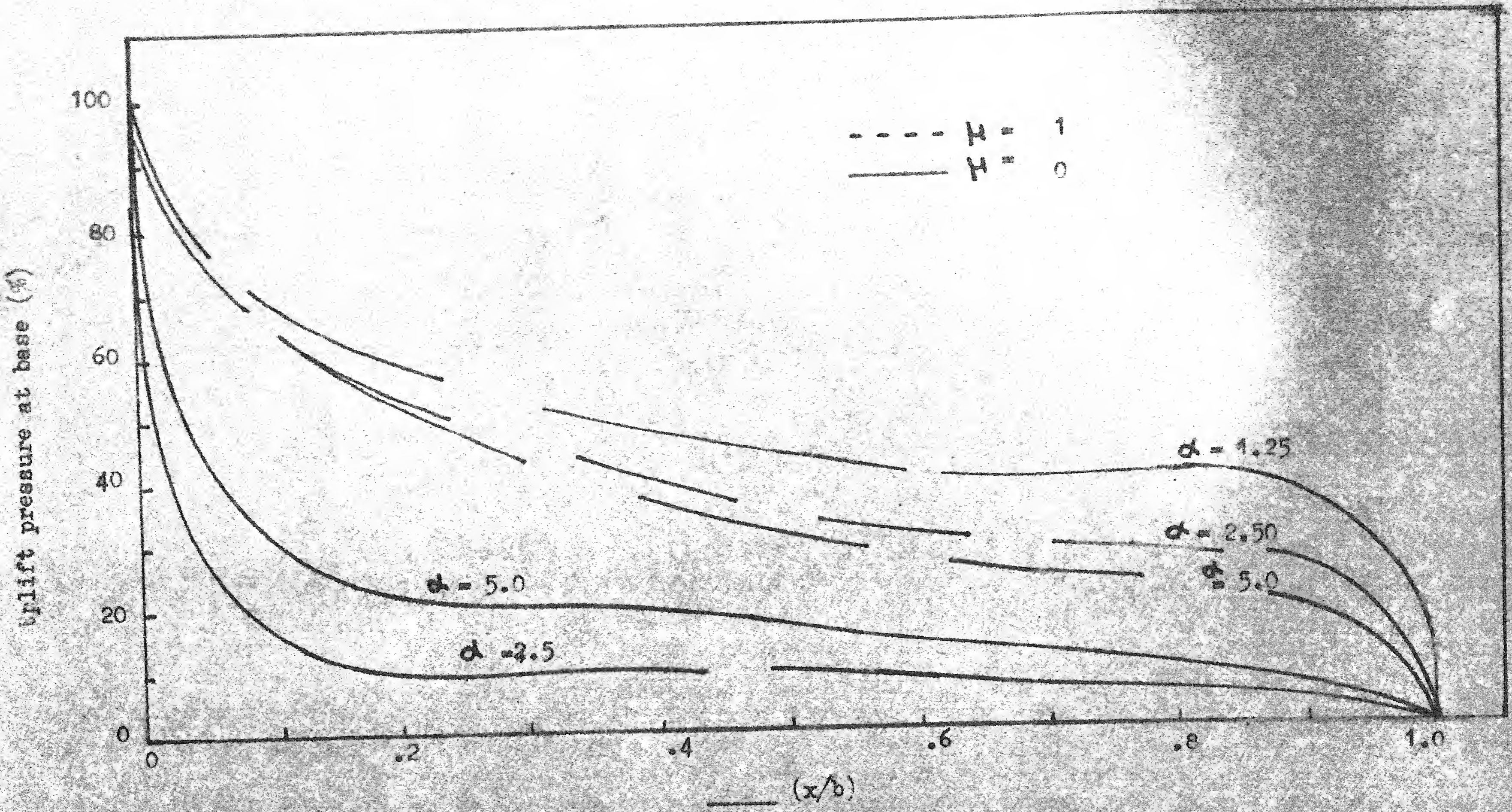


Fig. 5.35 Variation of uplift pressure at the base of $\theta = 60^\circ, \gamma = 60^\circ$

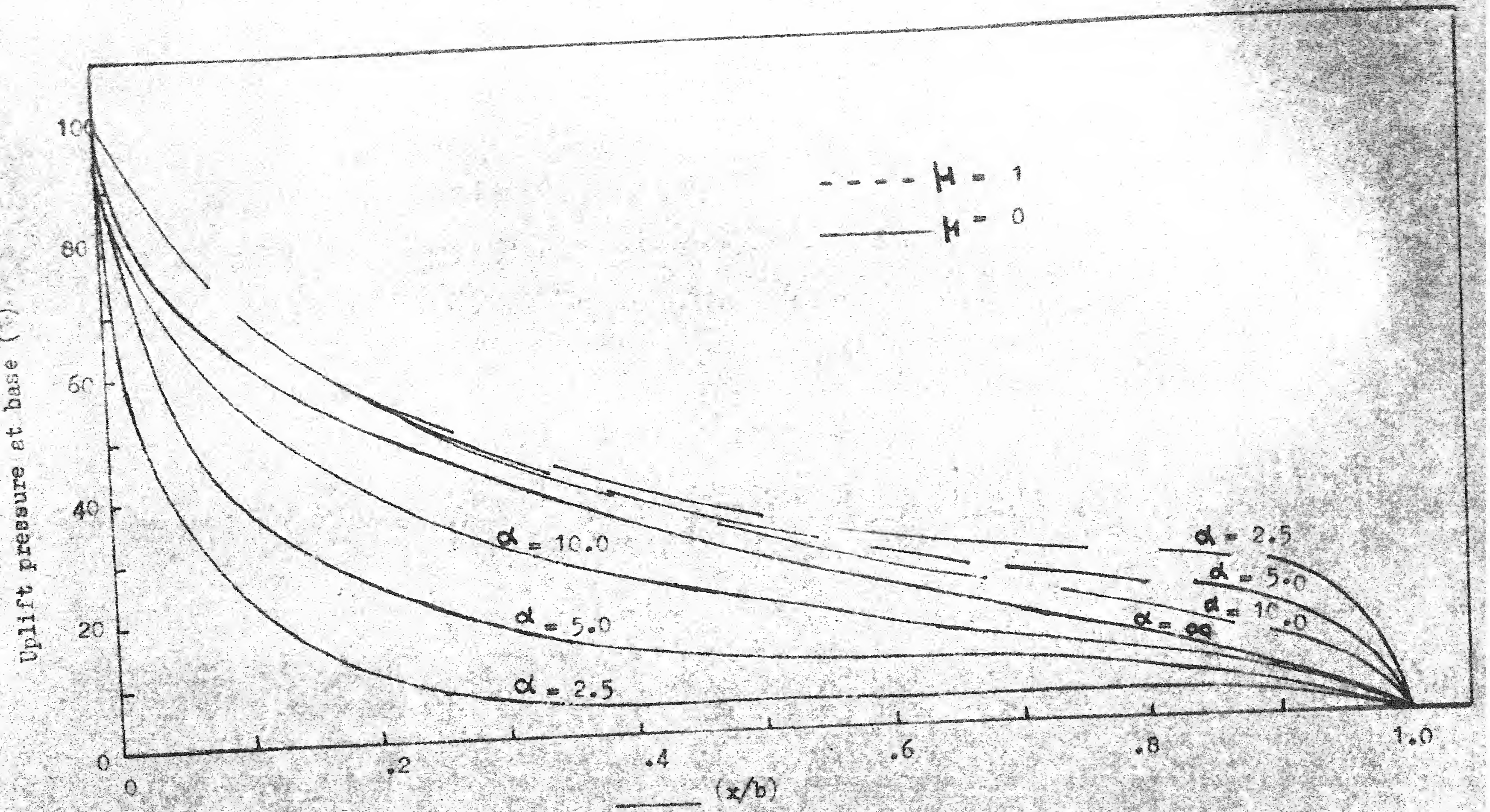


Fig. 5.36 Variation of uplift pressure at the base for $\theta = 60^{\circ}, \gamma = 90^{\circ}$

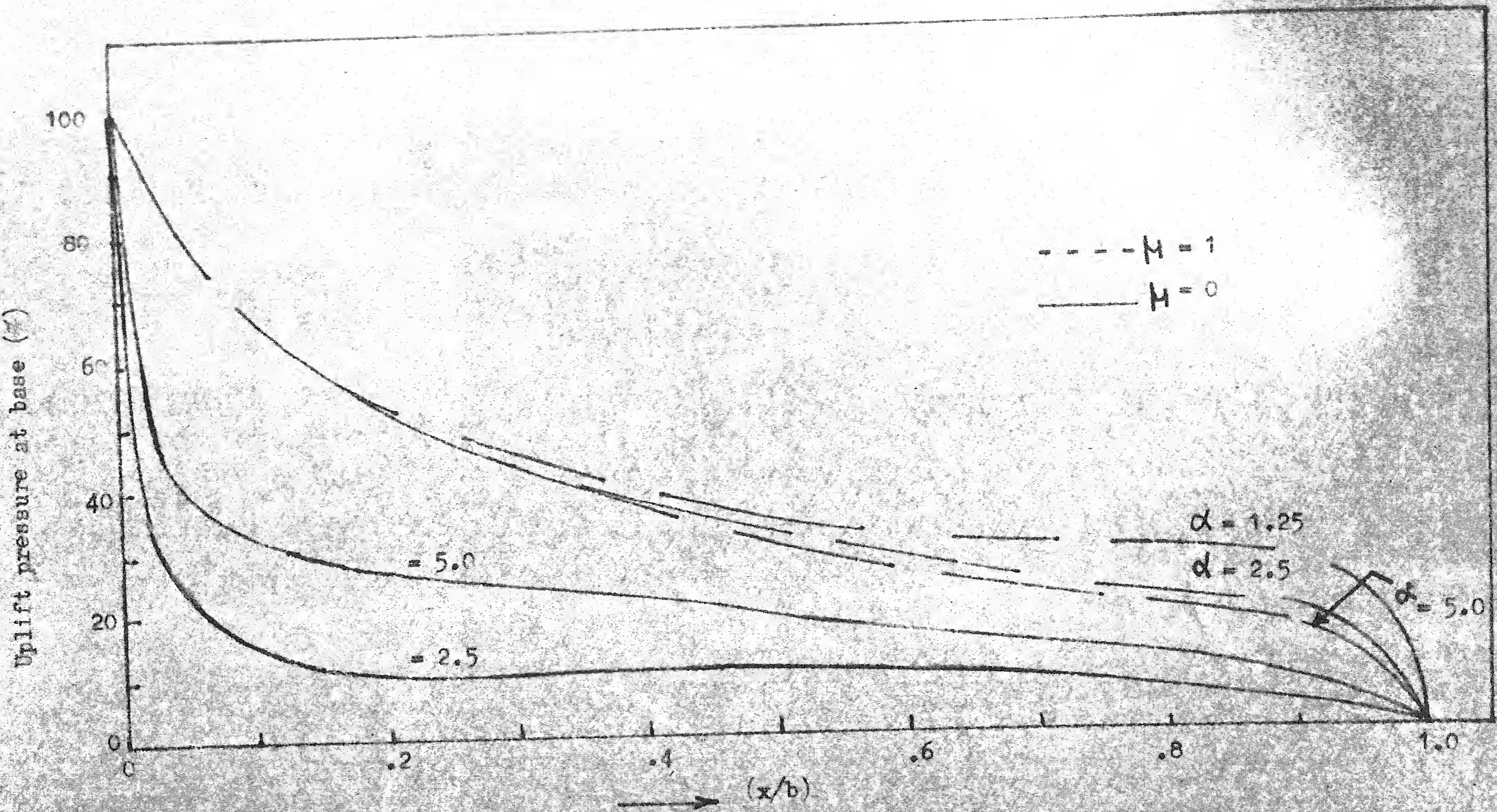


Fig. 5.37 Variation of uplift pressure at the base for $\theta = 60^\circ$, $\gamma = 120^\circ$

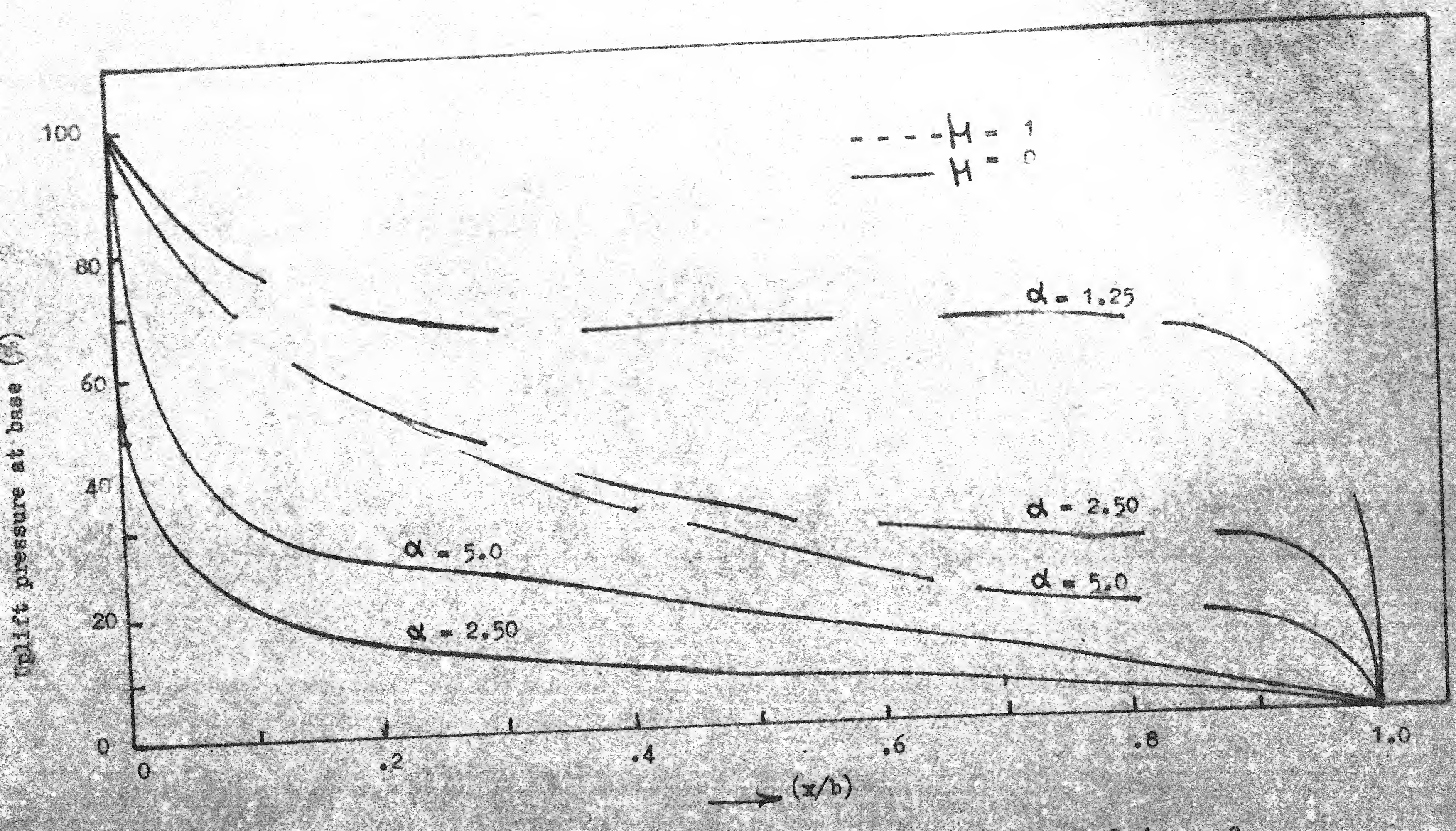


Fig. 5.38 Variation of uplift pressure at the base for $\theta = 30^\circ$, $\gamma = 30^\circ$

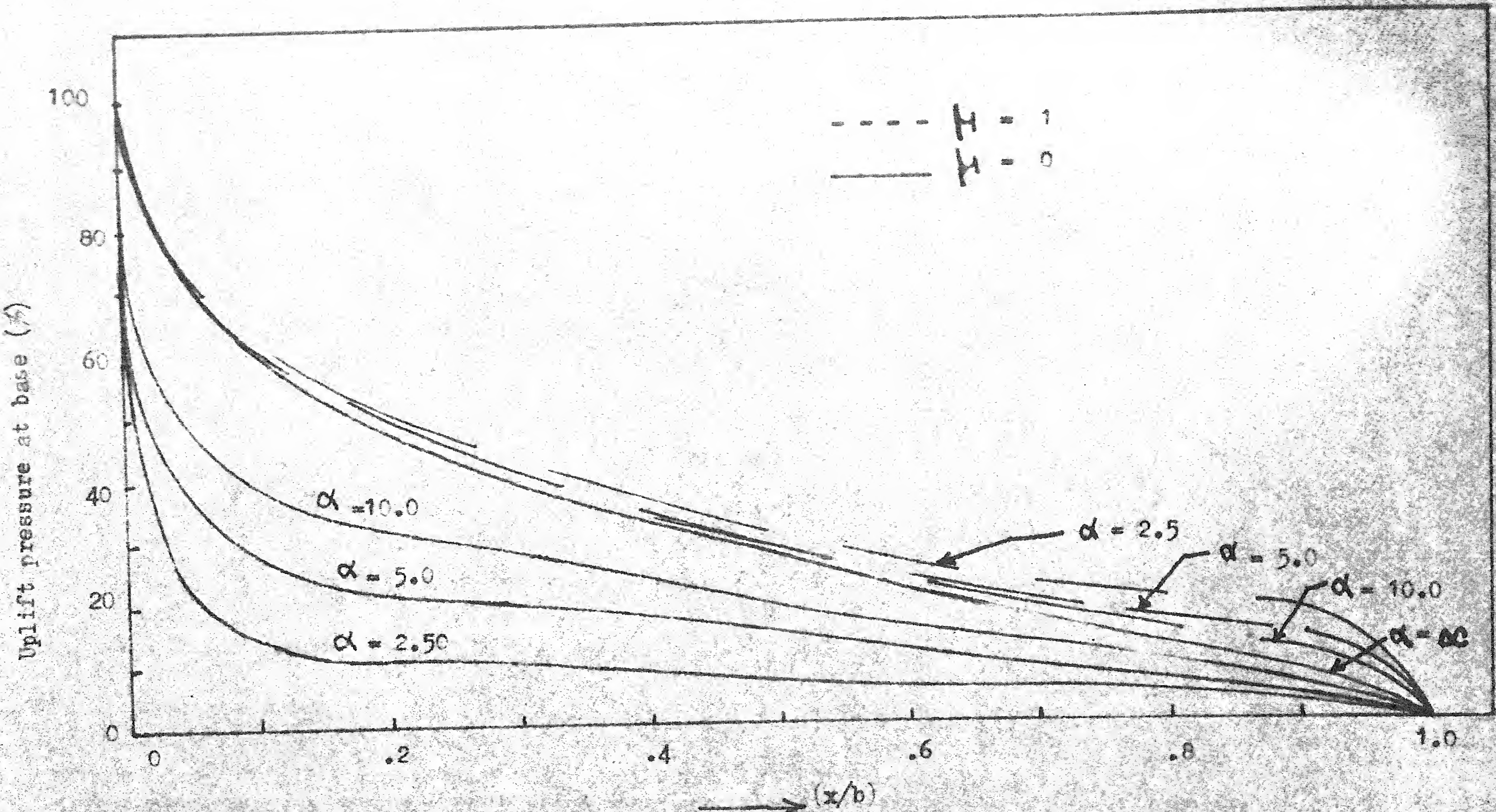


Fig. 5.40 Variation of uplift pressure at the base for $\theta = 30°$, $\gamma = 90°$

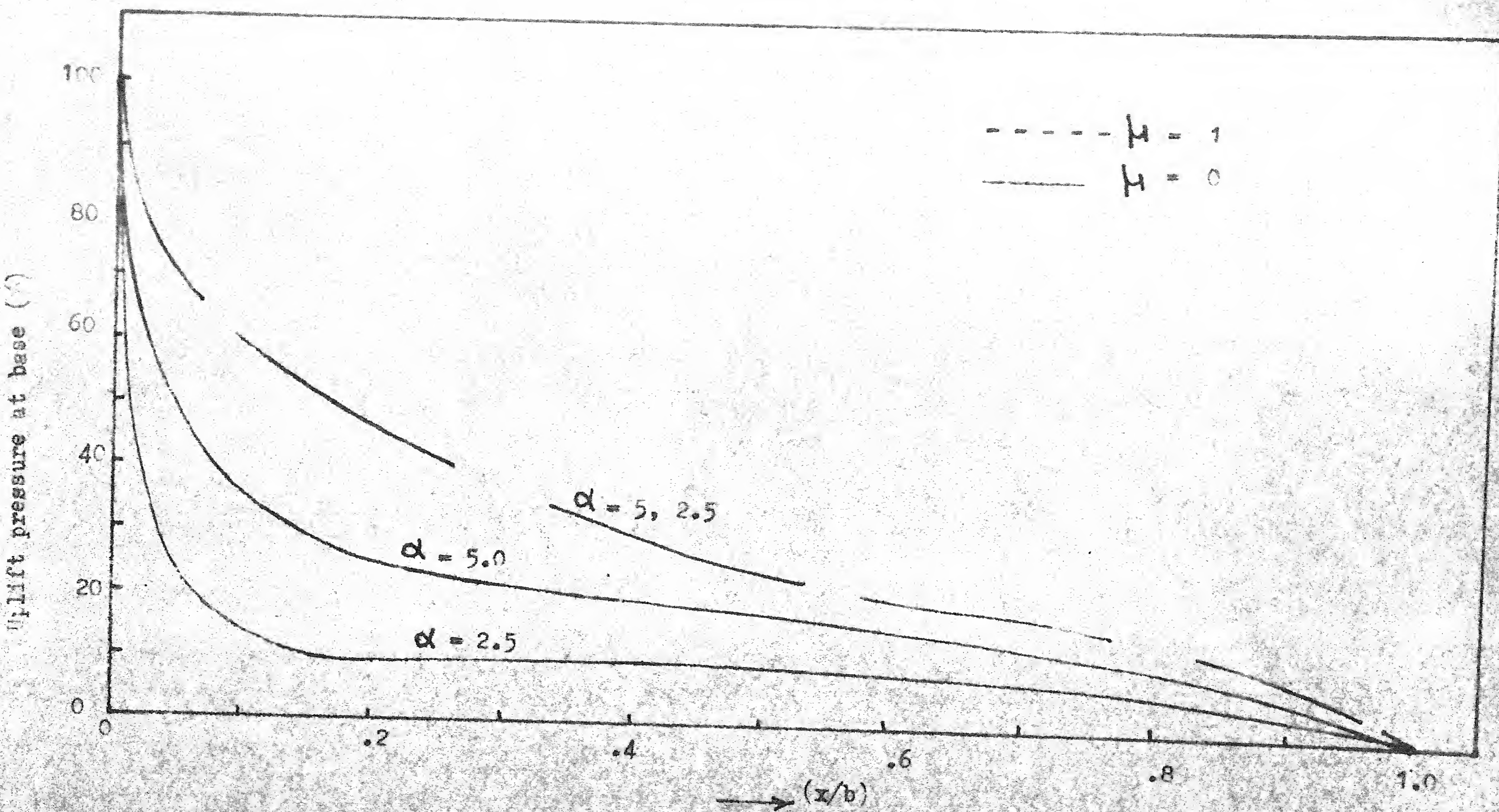


Fig. 5.41 Variation of uplift pressure at the base for $\theta = 30^\circ$, $\gamma = 120^\circ$

CHAPTER - 6

GENERAL CONCLUSIONS

Based on the studies reported in Chapters 4 and 5, the following broad based conclusions are arrived at:

(i) It is seen that by adopting few modifications to the standard electrical analogy methods the problem of confined flow through rock mass can easily be solved. The secondary permeability due to flow of water being easily simulated by drawing a line of silver paint on teledeltos paper.

(ii) The results indicate that the uplift pressure is influenced considerably by the presence of either vertical or horizontal joints. The degree of influence of joint being dependent on the disposition of joint in relation to the base of the weir and location of sheet pile.

(iii) The pressure head immediately next to sheet pile located at upstream end is considerably reduced when the horizontal joint is located near the base of the weir. However, there is not such a marked influence of joint on pressure distribution immediately next to sheet pile when the pile is located at the center of the weir (Fig. 4.14 and 4.15).

(iv) The pressure head at the tip of the sheet pile is reduced because of the major horizontal joint near the base of the weir (Fig. 4.10).

(v) The pressure head immediately next to sheet pile is seen to be considerably dependent upon the relative position of sheet pile and the joint along the base of the weir (Figs. 4.27 and 4.28).

(vi) It is seen that the anisotropy of the soil as well as the nature of the lower flow boundary have considerable influence on the uplift pressure distribution (results of Chapter 6).

(vii) The results obtained for various lower flow boundary conditions can serve as lower and upper bounds for a two-layered soil system separated by a boundary corresponding to the geometry of the lower flow boundary.

Because of the versatility of the electrical analogy method problems of confined and unconfined flow through the jointed rock mass for many practical situations can be analysed. Also the present study can be extended for the case of confined flow below weir with multiple sheet pile resting on anisotropic soil.

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